THE

PRINCIPLES

OF

ACTION in MATTER,

THE

GRAVITATION of BODIES,

AND THE

MOTION of the PLANETS,

explained from thoſe PRINCIPLES.

R

By CADWALLADER COLDEN, Eſq;

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BRITISH MUSEUM

TO THE

RIGHT HONOURABLE

The EARL of *Macclesfield*.

THE Deference that is juſtly due to your LORDSHIP's Opinion in regard to the Subject diſcuſſed in the following Sheets --- induces the Author with the utmoſt Reſpect to inſcribe them to you, being

Your LORDSHIP'S

moſt humble Servant,

Cadwallader Colden.

The PREFACE.

A *Few copies of the* two *firſt chapters of this treatiſe were publiſhed at* New York, *with deſign to know the ſentiments of the learned on new principles in natural philoſophy or phyſics, which were advanced in that eſſay. The* London *edition was without the author's knowledge, and had never been made could he have prevented it. Since that time he hasbeen incouraged to go on In this edition the two firſt chapters are reviſed, the matters contained in them follow more conſequentially, ſome obſcurities are removed, the ſentiments, it is hoped, put in a clearer light, and ſome new theorems added. In the following chapters, which were not before publiſhed, theſe principles are applied to the explication of the motion of the planets, and of the phenomena thereon depending, and of ſome other general phænomena, the cauſes of which, the author thinks, have not been before diſcovered. Authors are apt to be vain of their own conceits, and many are fond of novelties: it is for this reaſon, that many romantic theories in phyſics have made a glaring ſhew, like a meteor, and in ſhort time no tract of them remains. This, probably, has been occaſioned by ſome general and ſuperficial agreement, which theſe notions had with ſome remarkable phænomena; but when they came to be compared with accurate obſervation or experiment, with that preciſion which is neceſſary, the abſurdity ſoon appeared. For this reaſon the author has taken all the care in his power, to compare his theorems with the moſt accurate obſervations of aſtronomers, of which he propoſes to give ſome inſtances in the concluſion of this work.*

The doctrine of the mutual attraction of matter had in it ſomething ſo unphiloſophical, ſomething ſo like the occult qualities, which had been exploded, that nothing could have made it paſs with the learned, but the accurate agreement which *ſir* Iſaac Newton *ſhew'd it had with the phænomena. However juſtice muſt be done to this great author, that he no where calls it a* real *attraction, only* apparent, *the cauſe of which we know not. In this tract the author preſumes to think, that he has diſcovered the cauſe of this apparent attraction, and from which all the phænomena in gravitation evidently follow, as neceſſary conſequences: and that he has likewiſe diſcovered an error, which had ſlipped from the ſagacious ſir* Iſaac, *by his not knowing the cauſe of this apparent attraction,* viz. That the force of attraction is every where in proportion to the quantity of matter, *which in the enſuing work will be ſhewn to be otherwiſe,* viz. *that an equal quantity of matter, in different bodies, may have different powers of attraction; ſuppoſing that to be no ways affected or influenced by the attraction of any other body, or abſtractedly from their ſituation, in reſpect to any other body.*

Sir Iſaac Newton *no where gives the cauſe of the motion of the planets, but only ſuppoſes a certain degree of velocity to have been impreſſed upon them: in conſequence of which no reaſon is given for the moſt general and obvious phænomena of the motion of the planets, as particularly for the different diſtances at which the planets ſeverally, and the comets revolve, and the different excentricity of their orbits. The author pretends to have diſcovered the true cauſe of the motion of the planets and comets, and from thence to deduce the reaſon of all the phænomena, with that exactneſs as to agree with the moſt accurate obſervations, and even to point out ſome inequalities, from their cauſes, which have not been diſcovered by obſervation, by reaſon of the ſmallneſs and imperfection of aſtronomical inſtruments in common uſe.*

In the laſt place a method is ſhewn, how, with a competent ſkill in aſtronomy, equations may be formed for the motion of the planets, without any aid of the conic ſections, or of any other knowledge, beſides the common rules of arithmetic and

and trigonometry; of which an example is given, in a table of equations for the motion of the earth or apparent motion of the ſun.

As to other parts of this work, which are conjectural or phyſical matters, and given only as hints to thoſe who may incline to make a more particular and juſt enquiry, it is thought they do not deſerve a formal introduction: they are ſubmitted to the generoſity of the reader for their reception.

Though the author has preſumed, in ſome material points, to differ in opinion from the great ſir Iſaac Newton, *and to point out ſome errors he has fallen into, (and what man never fell into any error?) yet no man can have a greater opinion of ſir* Iſaac's *wonderful ſagacity and accuracy in diſcovering the moſt hidden truths, than the author has: this work it ſelf will ſhew what great advantages has been made of ſir* Iſaac's *diſcoveries.*

The author has had more in view to convey his conceptions clearly to others, than to any elegance of ſtile, or to the pomp of a demonſtrative method: and as he has been conſtantly employed in buſineſs, and could not purſue theſe ſtudies otherways than by way of amuſement, by fits and ſtarts, whereby the chain of thought neceſſary in ſuch a purſuit, has been often interrupted, he begs the reader's indulgence, in making allowances for the imperfections, which this muſt have often occaſioned.

A little eſſay wrote ſome years ago upon another occaſion, to explain the nature of infinite ratios, is deſigned to be annexed; as it may give a more compleat conception of what is intended, in ſeveral parts of this treatiſe.

It may be proper, by way of introduction, to give a general idea of the principles on which the following is founded; that thereby the reader may more eaſily conceive, as he reads along, what the author has in view.

PREFACE.

It is laid down as a principle, that all the primary or ſimple ideas we have of things external to us, ariſe from the impreſſions or actions of theſe things on our ſenſes: and, therefore, that the properties and qualities of things are nothing elſe but their various actions, or modes of acting, either ſimple or complicated. That the knowledge we have of things is no other than the perception of theſe actions, of their different degrees and different modes, and of the ratios of theſe differences to each other.

That all ſimple beings or things have one ſingle action, or manner of acting, eſſential to them; without which we have no conception of that thing.

~~Is power a mode of action?~~

That there are two, and perhaps only two, eſſential different modes of action, in material beings. The one a power by which the thing in which this action ſubſiſts does reſiſt all change of its preſent ſtate: the other a power by which the thing in which the action of moving ſubſiſts is continually changing its preſent ſtate, or ſituation, by motion, and gives motion to every other thing which at any time moves.

~~No body can — but strictly speaking no body can act at all: — The term action does not belong to body.~~

It is a ſelf-evident propoſition, that nothing acts where it is not: therefore, if any thing exert any action at a diſtance, this action muſt be communicated to that diſtance, by ſome medium from the place of the acting thing, to the place where the action is communicated. The mutual apparent attraction of bodies, at a diſtance from each other, ſhews the neceſſity of the exiſtence of ſuch a medium. This medium makes a third kind of matter, eſſentially different from the other two, by its equally receiving the action, or manner of acting, either of the reſiſting, or of the moving power, and by its reacting thoſe actions with the ſame degree of force or action it received them. From the nature of this medium, (commonly called æther) or from the neceſſary conſequences of receiving and reacting theſe contrary modes of action, the apparent mutual attraction of bodies at a diſtance from each other, and gravitation

vitation is explained, and the ſeveral phænomena thence ariſing.

Every thing, to which any action is eſſential, muſt exert that action equally in all directions; becauſe nothing can be conceived in the thing itſelf to hinder it, in one direction more than in another: then the direction of motion in the moving power, towards any one point more than towards any other point, muſt be by ſomething external, by the reſiſtance in that particular direction being leſs than in any other.

That which acts equally in all directions acts in none. that is does not act at all.

~~There is a mistake in thinking Direction applicable to any other kind of action than motion only: now nothing can possibly be supposed to move in all directions.~~

~~very applicable to elastic particles~~

Several arguments are produced, in this eſſay, to demonſtrate, that light is the ſubſtance or thing to which the power of moving is eſſential: and to thoſe therein mentioned, among which the principal is the demonſtrating in what manner the motion of the planets and comets ariſe from thence, this other argument may be added, that we can have no conception of light without motion, of which any one may convince himſelf by a proper attention. For example, if light be ſuppoſed to be compoſed of ſmall globular bodies at reſt, this ſuppoſition gives no idea of light or colours, it conveys no idea of any thing in common with the ideas raiſed in our mind by the action of light.

Stuff

It is expected, that the great authority which ſir Iſaac Newton *has juſtly obtained, will give a ſtrong prejudice, and perhaps for ſome time not to be ſurmounted, againſt the introductory part of the third chapter: wherein it is denied, that if a planet loſe its motion, by its gravitation in moving from the perihelion to the aphelion, and increaſe its velocity from the ſame cauſe only, in moving from the aphelion to the perihelion: I ſay, it is denied that, if the increaſe of velocity be from this cauſe only, that the planet by this increaſe of motion, be it never ſo great, can acquire any direction ſo as to make it recede from the ſun. For ſince the direction of the motion, by which the planet's velocity is increaſed, is towards the ſun, the greater the velocity, with more force*

~~This is a mistake~~.

it muſt move towards the ſun, and a greater force it muſt require to turn it out of this direction; but no other force is ſuppoſed to move it from the ſun, beſides the increaſe of velocity towards the ſun, by gravitation, nor any other force, to alter the direction; then the force of direction and the direction itſelf towards the ſun muſt continually increaſe, in place of growing leſs, or turning from the ſun.

The attractive force of the sun is not great enough to give a new direction but only to diversify ye old. — The author has neglected ye time & the velocity of the planets motion. See Ferguson's Astronomy

But if the motion of the planet be cauſed by the emiſſion of light from the ſun, then this chapter will ſhew how the direction of the motion of the planets comes to alter, both in the aphelion and perihelion. It will be ſhewn, that by the two contrary actions, viz. of light from the ſun, and apparent attraction towards the ſun, there muſt be a certain diſtance from the ſun, at which theſe oppoſite actions are equal: at which diſtance if the planet were not endowed with the power of reſiſting, or of continuing any action which it receives, it would continually move in a circle round the ſun. But as the planet, by its reſiſting power, continues any motion it receives, it gains a motion thereby of the nature of a projectile motion, by which it will recede from the ſun, and accede towards the ſun by turns, with a kind of oſcillatory motion; the center of which motion is in the circle of the planet's mean motion, and is ſuppoſed to move in that circle, ſo as to be always in the line connecting the centers of the ſun and planet. An idea of this projectile and oſcillatory motion in the planet may be conceived, by ſuppoſing a ball to be projected perpendicularly to the horizon, and a hole being made through the center of the earth, that this ball, by the force of gravitation and the velocity it acquires thereby, in its return paſſes quite through the earth, and aſcends on the oppoſite ſide of the earth, preciſely as far from the earth's center as it did by its firſt projection, and continues thus perpetually to oſcillate. The

The motion of the planet is certainly caused by the emission of light or vibratory motion propagated from the Sun; but this makes no difference.

principal difference in the perception of theſe two oſcillatory motions of the planet and ball, is, that in that of the ball the center of oſcillation is imagined to be at reſt, in the other the center is continually moving forward in the circumference of a circle of which the ſun is the center.

It will be ſhewn how a planet acquires this projectile motion, that this motion added to the centripetal motion from gravitation, is preciſely equal to the centrifugal motion from the action of light in the perihelion; therefore that the planet cannot approach nearer to the ſun, and that in this point the planet's motion in its orbit is perpendicular to the line connecting the centers of the ſun and planet: in like manner that the centrifugal force from the emiſſion of light added to the projectile force, is preciſely equal to the centripetal force in the aphelion; therefore, that the planet cannot recede farther from the ſun, and that the planet's motion in its orbit is again, in this point, perpendicular to the line connecting the centers of the ſun and planet. Again, for the ſame reaſon that the motion of the projectile, as before ſuppoſed, is the ſwifter the nearer it is to the center of the earth, the velocity of the planet's projectile motion will be greater the nearer the planet is to the circle of its mean motion: and then the direction of the planet's mean motion in its orbit will make the moſt acute or obtuſe angle with the line connecting the centers of the ſun and planet, or the planet will then go with the greateſt velocity from its circular motion. For though the centripetal and centrifugal forces would be equal at the planet's mean diſtance, were the planet not endowed with the power of continuing every impreſſion or action it receives, yet, from this power the projectile force added to the centrifugal in the planet's receding from the ſun, and added to the centripetal, in the planet's acceding towards the ſun, makes the greateſt difference between the centripetal and *centrifugal actions at the planet's mean diſtance from the ſun.*

From theſe principles, the author thinks all the phænomena of the motion of the planets may be well accounted for, and which cannot be done on any other principles hitherto adopted into philoſophy. However he may have failed, in the method which he has taken to convey his conceptions to others, or in the application of theſe principles to ſome particular phænomena, yet he is perſuaded, from the repeated reflexions he has had on this ſubject, for ſome years paſſed, that ſome able hand, from the hints here given, will ſet theſe principles in ſuch a light that they will univerſally prevail, and thereby a new light be given to every part of natural philoſophy, ſo as to make conſiderable improvement in every branch of phyſics. The author hopes that it will be of uſe, for theſe purpoſes have been the principle motive to the preſent publication.

THE
PRINCIPLES
OF
ACTION in MATTER, &c.

CHAP. I.

Of the Principles of Action in Matter.

SECT. I.

Of the eſſential Properties and Differences of Things.

1. WE have no knowledge of ſubſtances, or of any being, or of any thing, abſtracted from the action of that thing or being. All our knowledge of things conſiſts in the perception of the power, or force, or property, or manner of acting of that thing; that is, of the action of that thing on our ſenſes, or of the effects of that

thing on ſome other thing, whoſe action affects, or is the object of our ſenſes, and in the perception of the relations or ratios of theſe actions to each other. For if any thing produce no alteration in our ſenſes, it is impoſſible for us to know that any ſuch thing exiſts: and every effect muſt be produced by ſome cauſe, or by ſome action.

2. Every thing, that we know, is an agent, or has a power of acting: for as we know nothing of any thing but its action, and the effects of that action, the moment any thing ceaſes to act it muſt be annihilated as to us: we can have no kind of idea of its exiſtence. Any thing, which produces any effect or alteration in another thing, muſt either have its power and force of itſelf, or its action muſt be the effect of ſome other cauſe, and conſequently it is not a primary agent: but in this place I only conſider primary agents, and ſuch muſt continually exert their force and energy. For it ſeems a contradiction in terms that any thing ſhould obſtruct or oppoſe its own force or energy: conſequently every thing exerting of it ſelf any power or force muſt be continually acting. Whatever ſtops be ſuppoſed to happen in its action, they can only be the effects of ſome oppoſite power or force, acting at the ſame time, and of ſuperior or equal force in that inſtance, by which, tho' it continue to exert its power, its action is ſtopt, or rendered ineffectual.

3. If the idea or conception, which the action of any thing excites in us, be ſuch, that at the ſame time we per-

ceive that the thing itſelf, endowed with this power of acting, muſt be of ſome ſhape, or that it may be divided into parts, that its power or force may be increaſed by addition of parts, or leſſened by the taking away of part, it is then conceived to be of ſome quantity. And every thing which is conceived to be of ſome quantity, is commonly called *matter*.

4. But if the action of any thing do not excite in us any perception of its being of any quantity, or of any form or ſhape, or of its conſiſting of any parts, or that its force or action can be increaſed or leſſened by the addition or taking away of part, that thing commonly goes under the name of an *immaterial* ſubſtance, or of *ſpirit*.

a kind of
contradiction in
terms?

5. Now from this manner of conſidering things, I hope to ſhew, that we may have as clear and diſtinct an idea or conception of ſpirit, as we can have of matter: and that all the difficulties or abſurdities, which many have fallen into, ariſe from an error in the conceptions of the power, force, energy, or manner of acting, and which are commonly called the properties or qualities of things; or from the confuſion which ariſes by the uſing of different terms, or words to expreſs things, which in themſelves are not different. The property, or quality of any thing, is nothing elſe but the action of that thing: and the different qualities or properties of any thing or ſubſtance are no other than the different actions or manner of acting

of that thing. For we can have no idea or conception of the property or quality of any thing excited in us, but by the action of that thing: unleſs it can be ſuppoſed, that an effect is produced without a cauſe, and that a cauſe can be conceived without action. Therefore,

6. The differences of things (ſo far at leaſt as we can know) conſiſts in their different actions, or manner of acting. And if the actions of ſeveral things be ſuch, as that we evidently perceive, that they cannot proceed from the ſame power or force, or kind of power or force; ſuch things are ſaid to be in their *nature* different, or *eſſentially* different. The eſſence of things or of ſubſtances, ſo far as we can diſcover it, conſiſts in the power, or force, or manner of acting of thoſe things.

It remains then, in the firſt place, to conſider the ſeveral powers of thoſe things, which we conceive as conſiſting of ſome quantity, and which are commonly called matter. Afterwards the action of the immaterial powers, or of intelligent beings, may be conſidered.

SECT.

SECT. II.

Of the force of Reſiſting, or Vis Inertiæ *of Sir* Iſaac Newton.

1. THE moſt obvious property or quality in things which are the objects of, or affect our ſenſes, is that property by which they reſiſt or affect our ſenſe of touch; and which is ſometimes called the tangible quality of things. This was firſt conſider'd as a force or power by Sir *Iſaac Newton*, under the Name of *vis inertiæ*, or that power or force by which any thing reſiſts or oppoſes any alteration of the ſtate in which it is: and it is ſo generally obſerved, that it is commonly accounted a permanent eſſential property of all matter.

There are two
kinds of resistance
the resistance to
motion & of
resistance to
compression

2. That this is a real or poſitive power or force, is evident from theſe obvious and conſtant obſervations of the manner of exerting its power. Firſt, this power is ſometimes greater, at other times leſs; it may be augmented by an addition of more force, or of more parts, and it may be leſſen'd by taking away of part: for example, a ball of twenty pounds weight reſiſts a greater force than a ball of ten pounds, or of one pound. But a meer negative power, or paſſive power, as ſome call it, that is, an unactive power, cannot be augmented or leſſened; becauſe it is a negation of all power and action, and is truly a *non entity*. And next, the thing or being which exerts this force, is

truly an agent or power, whoſe principle of action is in itſelf independent of the force or action of any other thing: becauſe its force is exerted in oppoſing and leſſening the force and action of every other thing, and therefore cannot receive its force and action from that which it oppoſes or reſiſts.

3. Nothing of motion enters the conception of the action of this power. For in the motion the action is directed from one point to another, and its force or action is exerted in one only direction: it has no force in the oppoſite direction, or in any other line of direction, except in one. If then in the action of reſiſtance, or of the *vis inertiæ*, there were any kind of motion, it would reſiſt motion only when their directions are oppoſite, and could not reſiſt when their directions tend towards the ſame point. But the reſiſting power oppoſes motion equally in all directions, and therefore we cannot conceive any thing of motion, in the action of reſiſting. And this power equally reſiſts motion, whether itſelf be put in motion by any other power, or be at reſt. For if two quantities of the reſiſting matter or power be put in motion, and the one be greater, or reſiſts more than the other, when both at reſt, that quantity which reſiſted moſt when at reſt, will likewiſe reſiſt moſt when in motion. It is evident from conſtant obſervation, that motion no way encreaſes or leſſens the force or action of reſiſtance, and conſequently that it does not conſiſt in motion.

4. The thing, ſubſtance, or being, endowed with the power of reſiſting, or the *vis inertiæ*, is truly an agent, exerting a certain kind of action, whereby it perſiſts in its preſent ſtate, and oppoſes and leſſens the action of every other agent that can change that ſtate./ It exerts its force equally in all directions, and in a manner peculiar to itſelf, different from all other natural agents or powers.

5. Power, force, and action, are ſo eſſential to the conception of reſiſting, that the denying of it neceſſarily includes a contradiction in terms. For the power of reſiſting either does ſomething or nothing; if it do any thing, it acts; if it do nothing, it does not reſiſt in contradiction to what is ſuppoſed. Therefore that reſiſting is acting, may be reckoned among the ſelf-evident maxims, which cannot be denied without falling into an abſurdity.

6. Notwithſtanding this, we are ſo accuſtomed from our infancy to join motion to the idea of all action, that it is difficult for us to conceive any kind of action without it. But a little reflection may diſcover, that this is occaſioned by a continued faulty connection of ideas, which ought not to be joined together. When a man thinks, he certainly does ſomething; then thinking is acting, or is a kind of action: but this action cannot be conceived as either moving or reſiſting, it is a kind of action of a peculiar kind, differing from all other kinds of action.

No! No! No! No!

There's a knock-down blow

7. This kind of reaſoning, however, is by ſome not thought ſufficient, unleſs the *modus*, or manner of acting in the reſiſting power, be explained; otherwiſe, it is thought the words *reſiſting power* are words without a meaning, and ſignify no more than an occult quality, a deceitful or cunning covering of our ignorance. On this occaſion it is neceſſary to obſerve, that though I cannot explain the manner of acting when I think, yet I hope none will deny that I think: but I ſay further, we cannot explain the action of any ſimple power, no not the action of moving, though no man doubts of his knowing what motion is. When the moving power puts any quantity of matter into motion, what conception have we of its manner of acting? If it be ſaid to be by puſhing or preſſing the thing which it moves, I muſt aſk again, what idea have we of puſhing or preſſing different from moving? Have we any other idea, than that the moving thing communicates the action of moving to the thing which it puſhes, preſſes, or moves? Till the action of moving be explained by ſome other action than that of moving, I ſhall preſume to ſay, that we have as clear and diſtinct a conception of the manner of acting in reſiſting, as of the manner of acting in moving: that is, that the reſiſting thing communicates its action to the thing on which it acts. We have no idea of any thing but of action, and all ideas ariſe from the communication of ſome kind of action to the thinking being: ſimple ideas ariſe from the

action of ſimple powers, and complex ideas from the complicated actions of ſeveral ſimple powers. No ſimple idea can be explained; no definition or explication can give a blind man any idea of colours, or a deaf man of ſounds: and the explication of complex ideas is, the ſhewing of what ſimple ideas they are compounded.

8. It is generally concluded, that the force of reſiſtance is in proportion to the quantity of matter, or of the reſiſting thing, becauſe a greater quantity of reſiſting matter requires a greater force to move it. But it muſt be obſerved, that in conſidering the reſiſting power, there are two ways of conſidering it as quantity, *viz.* the quantity of extenſion or its bulk, and the quantity of its force. When we ſay two powers, producing the ſame effect, are equal, we then mean only the quantity of force; but two different quantities in bulk may have the ſame force. There may therefore be different ſpecies of this reſiſting matter, in which the proportions of their force to their bulk may be different: and as theſe different proportions may be infinite in number, the different ſpecies of reſiſting matter may be infinitely different. No experiment, which determines the force of two different agents to be equal, does, for that reaſon, determine that their bulk is ſo likewiſe. The abſolute force therefore of any quantity of reſiſting matter may be compounded of the degree of reſiſting, with which each particle is endowed, and the quantity or number of particles.

From Sir *Iſaac Newton*'s diſcovery of the infinitely different ſpecies of light, it is poſſible there may be as many different ſpecies of the reſiſting matter; and if there be, we may hope, that a method may be diſcovered to demonſtrate this. Before Sir *Iſaac*'s diſcovery, light was generally thought to be homogeneous: and yet he has ſo clearly demonſtrated the contrary, that none can doubt of its being the very reverſe, that it conſiſts of an infinite variety of ſpecies. Could any man have imagined, that this diſcovery, which had avoided all the curious reſearches of philoſophers to his day, was made by ſo ſimple and common a contrivance as a triangular glaſs priſm? Glaſs and priſms had been in uſe many ages, and in all that time it was never imagined that any ſuch diſcovery could be made. Who then can ſay what diſcoveries may be made, when ſuch another genius as Sir *Iſaac Newton*'s ſhall appear in the world?

9. Some object, that no agent or active power can be conceived void of intelligence, or that all active powers or agents muſt be intelligent, or conſcious of their own actions. I perceive no neceſſity of joining the ideas of acting and intelligence together, ſince they are ſeparate and diſtinct ideas. I obſerve, I think, every day multitudes of action, without that the idea of intelligence is neceſſarily connected with thoſe actions. They, who perceive the neceſſity of this connection, muſt ſhew it, and till that be done, we muſt delay the further conſideration of this objection, till

the nature of intelligent beings ſhall be conſidered and explained.

10. From the action of reſiſting we receive the idea of impenetrability; for by this action, the action of every other thing, by which it is ſuppoſed that it may be penetrated, is oppoſed: and as nothing of motion or change of place enters the idea of reſiſting, no other thing can be penetrated by the reſiſting power. How great the force of reſiſting is in the leaſt particles, which compoſe any quantity of the reſiſting matter, will be more clearly ſhewn afterwards.

SECT. III.

Of Motion, or the Moving Power.

1. WE daily obſerve that ſome things move, or paſs from one place to another: the power of reſiſtance is ſo very different from that of moving, that they can in no manner be conceived as the effects of the ſame agent, or of the ſame cauſe. We likewiſe obſerve, that ſome things in motion loſe their motion; that the ſame things again, or other things at reſt, acquire motion; and that things moving with a ſmall velocity, acquire a great velocity: this motion muſt then either be by ſome power, or force, or agency, in the moving thing itſelf, or it muſt be the effect of ſome other power or agent: this other thing muſt of itſelf be an agent, and has the power of

moving eſſential to it, or it muſt be put in motion by a third thing, and ſo on: we muſt then at laſt reſt in ſomething, to which the power of moving is eſſential, or we muſt allow that an effect can be produced without a cauſe. This thing then, to which motion is eſſential, which moves by its own natural power or force, muſt be an agent, which has its active principle in itſelf. When we ſee a ſmall ſpark gradually ſet a large city all in a flame, can any man imagine that there is no more motion in all the parts of the city thus in a flame, than there was in the ſmall ſpark which firſt began the fire? That there is no more power, or force, or action, in this prodigious fire, than there was in the ſcarce diſtinguiſhable ſpark which perhaps firſt began it? If there be not ſuppoſed ſomething mixed with the materials of the city thus all in fire, which has a power in itſelf of moving, all the prodigious force or action of motion in the city in fire, muſt be ſuppoſed in the little ſpark which firſt began the fire; for nothing can give what it has not. In like manner, when a ſmall ſpark ſets fire to a quantity of gun-powder, by which caſtles, and the greateſt weights are moved, the little ſpark cannot be ſuppoſed to have had in it all the force of motion, which is diſcovered from its effects to be in the gun-powder. Theſe, and innumerable other phænomena, evidently ſhew, that ſome things are ſelf-moving agents, and which ever move, unleſs hinder'd by the ſuperior force of reſiſting matter; and that as ſoon as the reſiſting power is removed, the ſelf-moving power immediately recovers its motion.

2. Tho' it be evident, from the idea we have of motion, that it can only exert its force in one direction, in paſſing in a ſtreight line from one point to another; yet it is equally ſuſceptible of any direction: and its aſſuming one direction rather than another, or its changing its direction, is only from the oppoſition of ſome reſiſting power. For this reaſon gun-powder in open air exerts its force chiefly upwards, becauſe in that direction it meets with the leaſt reſiſtance: but gun-powder, confin'd in a gun-barrel, meets with reſiſtance in every direction, except in that parallel to the bore of the gun-barrel, and therefore exerts its force in the direction of the bore of the barrel.

3. In conſidering motion purely of itſelf, abſtractedly from any other power or agent, we can conceive nothing but the degree of ſwiftneſs or velocity with which it moves, and the direction of its motion. Velocity, from the ſwifteſt to the ſloweſt we can imagine, may be divided into any number of degrees: ſo that as to its velocity it may be conſider'd as conſiſting of ſome quantity. But beſides this, we can conceive motion, or the moving thing, as confin'd within certain bounds, and conſequently of ſome ſhape or extenſion. It may then be divided into parts or particles: then if one particle, moving with a certain velocity, have any determin'd force or power, then two particles muſt have twice that force, moving with the ſame velocity, and

three particles thrice that force. So that the abſolute force of the moving agent or thing is compounded of the velocity, and the quantity of the moving thing.

4. Since then the reſiſting and moving powers are ſo contrary to each other in their nature, reſiſting motion and ſelf-moving, that it is impoſſible to conceive them both eſſential to the ſame thing, we muſt be careful then not to attribute theſe eſſentially different modes of action to the ſame agent: and ſince we cannot conceive impenetrability without reſiſtance, there is nothing in our conception of the action of motion which renders two quantities of the moving thing impenetrable as to each other; tho' as before obſerved, from the nature of the reſiſting power, the reſiſting and moving things muſt be mutually impenetrable to each other *. If then two quantities or particles of the moving thing were to meet each other in oppoſite or different directions, they not only could not ſtop each other's motion, but likewiſe could not alter each other's direction. For it is impoſſible to conceive how motion can ſtop or deſtroy motion, or that the direction of motion can be alter'd without reſiſtance; but reſiſtance, as before proved, cannot exiſt in the ſame power with motion, being contradictory to each other.

5. I am ſenſible how difficult it is to form proper conceptions of the action of the moving power, abſtracted from

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* Sect. 2. Par. 10.

from the reſiſting power, becauſe we commonly form our ideas of motion from reſiſting matter in motion. In order to remove theſe difficulties, let us ſuppoſe, that two quantities of matter meet, one of the moving agent, the other of the reſiſting what will happen. If the reſiſting power be greater or equal to the moving power, it will ſtop the action of the moving power, and both remain at reſt; tho' the force or endeavour to move ſtill remain in the moving power: but if the moving power be ſtronger or greater than the reſiſting, then the moving power will only loſe part of its motion, and both will move together with the remaining force of motion or velocity. This becomes a compound quantity, endowed with both motion and reſiſtance.

6. Again ſuppoſe, that this united quantity of the moving and reſiſting agents be by ſome means ſeparated, the moving matter will, the moment after it is ſeparated from the reſiſting matter, recover its firſt or original action of moving, or its whole motion; and the reſiſting matter will continue its motion in the ſame manner, or with the ſame velocity it did when joined with the moving matter, and this by its power of reſiſtance; for this power conſiſts in perſevering in whatever ſtate it is in, and in oppoſing all change of that ſtate. As the degree of velocity with which it moves is the ſtate in which every part of that reſiſting matter is in at that time, the force of reſiſting all change, or perſiſting in that ſtate in

two different quantities, moving with the ſame velocity, will be as the ſeveral quantities of reſiſting matter in each; and in different quantities moving with different velocities, as the reſpective velocities multiplied into their quantities of reſiſting matter. This force is by Sir *Iſaac Newton* called *momentum*, and is by him rightly diſtinguiſhed from motion or velocity: for it is a complicated, not a ſimple power, and ariſes from the joint action of agents eſſentially different in their natures and manner of acting. Any quantity of reſiſting matter, moving thus by the action of the moving power communicated to it, loſes the whole, or ſome part of its motion, every time it meets with any other quantity of reſiſting matter, and never of itſelf recovers that motion again. The laws of this communicated motion in reſiſting matter, being moſt common to our obſervation, are well deſcribed; but the laws by which the primary agents act are little underſtood, tho' they be the true cauſes of all the phænomena or appearances in nature, and therefore deſerve the trouble of further inquiry. I hope in another place more particularly to deſcribe the moving power, and to ſhew that there is ſuch a thing diſtinct from all other things, which come under the obſervation of our ſenſes. I hope to ſhew, that as the reſiſting power is the proper object or cauſe in our ſenſe of feeling, ſo the moving power is the proper object or cauſe of ſeeing; or as the tangible qualities of things ariſe from the reſiſting power, ſo the viſible qualities ariſe from the moving power.

Concurrence X

Agent

Reagent or Patient

SECT.

These are well Distinguished throughout this Sectn

SECT. IV.

Of the Elaſtic Power, or of Æther.

1. NOthing can act where it is not; and therefore nothing can communicate its action to any other thing, which is at a diſtance from it, but by the intervention or communication of ſome middle thing or *medium*, which extends from the one thing to the other. As the hand, for example, cannot communicate action to a ball it does not touch, but by the intervention of a ſtick, or other thing between the hand and the ball. We perceive that the ſun communicates ſome kind of action to all the planets, which circulate round it, by which they are retain'd in their orbits, or gravitate towards it: So likewiſe we obſerve, that the earth communicates action to bodies at a diſtance from it, by which they gravitate towards the earth. There muſt then be ſome *medium*, ſome middle thing, by which the ſun and the earth communicate action to other things, at a diſtance from them. This cannot be by an emanation of ſome kind of virtue, as ſome imagine; becauſe any kind of emanation ſuppoſes motion from the thing which emits the virtue to the thing on which the virtue exerts its force; but it is impoſſible to conceive that any motion from a thing, can cauſe a motion to that thing. If then we deſire to inquire into the cauſes producing the ſeveral phænomena or appearances in nature, it becomes

neceſſary to diſcover the nature of this medium, by which things communicate their action at a diſtance. It is called *æther* by Sir *Iſaac Newton* and other philoſophers; and all ſuppoſe the exiſtence of ſuch a thing.

2. The nature then of æther is to receive the action of any thing to which it is contiguous, and to communicate the ſame action to any other thing at a diſtance, to which it is likewiſe contiguous. And ſince the æther receives and communicates action at all diſtances, and every point of the æther muſt receive the action, and react, or communicate that action; as when the hand ſtrikes a ball by the means of a ſtick, every particle of the ſtick receives the action of the hand, and communicates it from the one end to the other.

3. Since æther cannot be conceived to act or receive action where it is not, however ſmall the diſtance be, all the ſeveral parts or points of æther which receive and communicate action muſt be contiguous to each other, or there cannot be the leaſt diſtance between the points of æther. This may not ſeem neceſſary in communicating motion, yet in communicating any other action, which has no motion, it is neceſſary that the parts be contiguous.

4. Conſequently the receiving action, to the greateſt diſtance that it extends, muſt be at the ſame inſtant in in every part or point, in the moſt diſtant as well as the neareſt. As we ſee the motion communicated at one end of a ſtick, or any other thing, whoſe parts are contiguous,

or not ſeparated, is in the ſame moment communicated from the one end of the ſtick to the other, whatever the length thereof be.

5. The æther receives the action of reſiſting matter, as well as moving, and communicates the action, which it receives, from every point to every thing contiguous to that point, by a kind of expanſive action proceeding in all directions from each point, as a center. And the force of expanſion or reaction, in each point, is every where obſerved to be equal to the force of action communicated to that point. As the idea of this expanſive power is commonly taken from elaſtic bodies, whoſe parts, if they by any force be compreſſed nearer to each other, reſtore themſelves to their former place, with a force equal to that which compreſſed them, the æther is frequently called an elaſtic fluid.

6. Since then the æther expands or reflects the action of reſiſtance in all directions, in a direction oppoſite to the direction in which it received the action of the reſiſting thing, as well as in the ſame direction, they cannot both act in the ſame inſtant, becauſe both their actions are reſiſting and equal in that inſtant, or deſtructive of each other's action, conſequently they muſt act by fits and turns, that is, while the reſiſting power is in action, the æther is in a fit of receiving action, or of inaction; and while the æther is in a fit of reaction, the reſiſting power is in a fit of inaction, alternately by turns; ſo likewiſe as the reaction of

reſiſtance in æther is deſtructive or negative to the action of motion, the moving thing muſt likewiſe act by fits, or by alternate turns of action and inaction. And ſince impenetrability is the conſequence or effect of reſiſtance only*, æther is only impenetrable during its fit of reacting reſiſtance, at other times it is penetrable by the moving thing which has no power of reſiſting. This is curiouſly obſerved by Sir *Iſaac Newton* in his Optics in the paſſage of light.

7. Now it is plain, that the elaſtic or expanſive force of the æther; cannot be the effect of the reſiſting agent: for the whole power of the reſiſting agent is exerted in its preſerving its preſent ſtate: but the idea of this includes a perpetual endeavour of change by a kind of expanſion. Again, the reſiſting power is exerted in oppoſing, leſſening, and rendering ineffectual all motion; but this power, by its reaction, preſerves the motion impreſſed upon it in its full force, and communicates the motion to every thing round it by expanding the action received in all directions. Neither can it be the effect of the moving power, for the moving power exerts its action only in one direction, but this power exerts its action in every direction from every point. This then muſt be a force or power eſſentially different both from the reſiſting and moving agents: for the effects of this power cannot be produced by either of theſe ſimply, or by both jointly. The whole manner of acting

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* Sect. 2. par. 10. and Sect. 3. par. 4.

in this power is ſingular and peculiar to itſelf; and yet it exerts no kind of action without the concurrence of ſome other power, and in their abſence and ceaſing to act it is in a ſtate of inaction; and though it in no manner receives its power of expanding or reflecting the action from any power, yet it imitates the manner of acting of every power whoſe action it receives.

8. The reſiſting power is negative to every other power, as well as the moving power, it leſſens the elaſtic power of the æther as to its reflecting or expanding motion; therefore in whatever degree the action of reſiſtance is communicated to the æther, in the ſame degree is its action of expanding or reflecting motion leſſened. But the moving and elaſtic powers being no way negative or reſiſting to each other, they no way leſſen each others action.

9. Elaſtic force has been uſually attributed to the ſhape of the parts which compoſe the elaſtic body. They are ſuppoſed to be ſpiral ſprings, like watch-ſprings. But why a ſpiral ſhape, or any ſhape, or arrangement of the parts of any thing, ſhould give it any power, which it had not before, is to me inconceivable. Common obſervation makes it manifeſt, that a ſpiral of lead is as little elaſtic as a ſtrait line of lead.

true because it
does not depend
of form —

10. Though this power, by which the æther expands or reflects, and conveys the action of any other power to any diſtance, be called elaſtic by Sir *Iſaac Newton*, and other philoſophers, yet the action of the elaſtic æther muſt not

Strange!

be conceived as in any manner ſimilar to that of elaſtic bodies, ſuch as a ball of ivory; but as a kind of action ſingular and peculiar to itſelf, and which cannot be explained by any ſimilitude to the action of any other thing or power, no more than the actions of reſiſting and thinking can be explained by moving. Therefore if one ſhould imagine that the æther conſiſts of innumerable ſmall globules (as of ivory) whoſe parts being preſſed together rebound with the ſame force with which they were compreſſed, he would have no conception of the elaſtic action which I mean. The actions of all firſt principles, and the ideas of them, muſt be ſimple, nothing of ſhape, or of parts, or of number, or of any thing like compoſition can enter into the conception of theſe ſimple powers; for otherwiſe they cannot be ſimple. In any conception of globules, they muſt be conceived as conſiſting of parts, which being preſſed nearer to each other, endeavour to ſeparate again: and as this cannot be conceived, without motion, ſuch elaſtic power can never reflect or continue any other action but that of moving. It is true, that in machines, and ſuch like aggregates, there is a kind of compound action, which none of its parts have ſeparately, and by this the machine, or aggregate, becomes a kind of unity (or τὸ ἓν, as the *Greeks* expreſs it) for its eſſence is deſtroyed by diviſion, and it no longer remains the ſame thing: but every one eaſily perceives, that no machine can be a ſimple being, or its action ſimple, but is the complication of ſeveral ſimple actions. The manner of acting in machines can therefore be explained by diſcovering the ſimple actions of which they are compounded: but the actions of ſimple powers, as before obſerved, cannot be explained.

11. Since the æther has no other action but what it receives, either from the reſiſting or moving power, and we have no idea of any thing but what we receive from its action, we muſt conceive æther as quantity, in the ſame manner we do reſiſting or moving matter; becauſe it can receive a greater or leſs degree of force or action from the reſiſting or moving agent; and conſequently its reaction, in which its peculiar power conſiſts, may be greater or leſs. If quantity then be the diſtinguiſhing character of matter, the æther muſt be a ſpecies of it.

12. We have before ſhewn, that impenetrability is a neceſſary conſequence of reſiſtance, * and therefore that the æther in the reaction of the reſiſting power muſt likewiſe be impenetrable, and that it muſt by fits and turns be penetrable and impenetrable †. It may then be objected, that, if the æther be thus by fits and turns penetrable and impenetrable, reſiſting matter muſt be ſo likewiſe, for, as it has been before ſhewn, that reſiſting matter exerts its action of reſiſtance by fits and turns. But if it be conſidered, that while the reſiſting power is in its fit of inaction, the æther is in its fit of action or reaction of the reſiſting action, then any

moving

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* Sect. 2. par. 10. † Sect. 4. par. 6

moving thing, by which action only any other thing can be penetrated, is ſtopped by the reſiſting action of the æther, and the moving thing can never act upon, or paſs through or penetrate the reſiſting matter, but while it is in its act of reſiſting, and conſequently can never paſs or penetrate it. It may be likewiſe objected, that we cannot conceive things to be different without conceiving them at the ſame time to occupy different places. In anſwer to this I muſt repeat, what has been ſeveral times before ſaid, viz. We have no idea or conceptions of ſubſtances or of things themſelves, nor have we any conception in what manner ſubſtances or things exert their action, our ideas are only excited by the actions of things on our ſenſes. If then any thing ceaſe to act in ſuch a manner as can oppoſe the action of another thing, there is nothing in our conception to hinder their being both in one place. So that truly we may conceive, that the reſiſting thing or the reſiſting matter cannot be in the ſame place with any other thing, becauſe it oppoſes the action of every other thing by which that thing can penetrate it. There is nothing in the action of moving to deſtroy motion, and therefore there is nothing in the conception of motion, without joining reſiſtance to it, to ſhew that two moving quantities cannot be in the ſame place. There is a great deal of reaſon to think, from the phænomena of light, that any quantity of light is penetrable by another quantity of light, or that the rays or particles of light are mutually penetrable by each other, every the leaſt particle of an opaque body reflects the rays of light, the diſtance between the points, which reflect light, is leſs than can be diſtinguiſhed by the beſt microſcope: there muſt then be above a million of theſe points in a ſquare inch. Every one of theſe reflect rays of light in all directions, and therefore they muſt croſs each other in innumerable places. Therefore every inch of a great room, for example, has innumerable points reflecting rays of light, which croſs each other. If then the rays or particles of light be impenetrable to each other, they muſt meet and oppoſe each others motions and change each others directions in innumerable places, by which ſuch confuſion muſt ariſe, that nothing could be ſeen diſtinctly. It is likewiſe obſerv'd, that tho' an infinite number of rays from all the points of a large burning glaſs, and that all of them interſect each other in the focus, yet the motion or direction of motion of not any of them is impeded or alter'd by their thus meeting and interſecting each other.

[illegible]

13. All the diſputes that happen among men, who have nothing elſe in view beſide the diſcovery of truth, ariſe from the faulty conceptions they have of things, by our ideas being not true or perfect images of the things they are ſuppoſed to repreſent: whenever the argument on both ſides is loaded with ſuch difficulties that we know not how to extricate ourſelves, it is from thence evident that we have not perfect conceptions of the things about which

it were ſufficient if
men had similar concepti
[illegible] expressed by the same
words

which the diſpute ariſes. Of this ſort is the diſpute about a *vacuum*, whether there can be any place in nature abſolutely void. If we can have no conception of an abſolute void, we cannot affirm that it is, or is not; and what conceptions can a man have of a place void of every thing, and of which nothing can be affirm'd? we cannot affirm any thing of it, for the moment we do, it muſt be ſomething, and if it be any thing, then that thing exiſts in that place, and the place is not void in contradiction to what would be prov'd. It is evident, from what was before ſaid, that all the parts of the æther are contiguous, or no void ſpace between them, except where their place is taken up by reſiſting matter, and if ſo there can be no *vacuum*. Sir *Iſaac Newton* and his followers on the contrary think there muſt be a *vacuum*, and their reaſon is, that all matter has the *vis inertiæ* or the force of reſiſting. If it were ſo, then the ſuppoſition of a *vacuum* would become abſolutely neceſſary; for without it there could be no motion. If all matter were equally endowed with the power of reſiſtance, as Sir *Iſaac* ſuppoſes, the ſuppoſition of a *vacuum* becomes neceſſary; but if it be true, as I think I have prov'd, that there are different ſpecies of matter, and that only one ſpecies has the power of reſiſting, and that this (as will appear upon the leaſt reflection) is by far the leaſt part of the univerſe, all the difficulties as to motion on the ſuppoſition of ſpace being every where full, vaniſh.

Jargon —

14. It follows then from the whole of what precedes, that the ſpecies of matter above deſcrib'd are agents, or acting principles: that each hath a power or force peculiar to itſelf, differing from the others in its eſſence and manner of acting. Whether there be any more ſpecies of matter is not eaſy to determine, tho' moſt of the ancients agree in this number. That theſe three are eſſentially different I can make no doubt. If there be any other ſpecies of matter, it muſt likewiſe be an active principle. For this reaſon ſome of the ancient philoſophers aſſerted that all nature is alive, that is, all nature is active. Try to deſcribe matter without power, force, or action, the whole deſcription muſt conſiſt of negatives, that is, it muſt be the deſcription of *nothing:* and then it very certainly follows, that nothing or no-being exiſts no where. The word *matter*, when it repreſents a meer paſſive being without power, force, or action, or property, is ſynonimous to the word *nothing*. Perhaps the chemiſts aim at the ſame thing I do, in the three principles they eſtabliſh, viz. *ſalt*, *ſulphur*, and *mercury*. By ſalt they may intend reſiſting matter, by the action of which the parts of matter (as may be hereafter ſhewn) are kept in union; by ſulphur they mean the moving matter; and by mercury the æther by which the action of other matter is (as by a meſſenger) carried to any diſtance, or becauſe, like *Mercury* or *Proteus*, it imitates the action of the

antients a pretty argument!

contradiction — or unintelligible.

the other ſpecies of matter. But it is beſide my preſent purpoſe to enquire into the opinions of philoſophers.

15. I ſhall only now add, that I ſuppoſe that this our earth, and all upon it, which affect our ſenſe of touch, the planets, and every thing commonly called body, to conſiſt chiefly of reſiſting matter: that all ſpace between theſe great bodies, and likewiſe the interſtices between the parts or particles which compoſe them, are filled with æther: that the moving matter (as light) is every where paſſing thro' the ſpaces fill'd with æther; conſequently that the ſpace fill'd with æther and light is vaſtly greater than the ſpace fill'd with reſiſting matter or body. If then I can ſhew, that the moſt general phænomena in nature, and ſuch as have hitherto puzzled the philoſophers of all ages, can be explain'd and made eaſy to our conceptions, from the actions of the different ſpecies of matter, as the cauſe of thoſe phænomena, it will be a further and ſtronger proof of what has been before advanc'd, and will at the ſame time ſhew the uſe of what I attempt to teach. I ſhall next proceed to explain ſome of thoſe general phænomena, and to ſhew how they ariſe from the actions of theſe different ſpecies of matter. But before I proceed, I believe the reader will not be diſpleaſed that I take notice of one thing, by which it will appear, that the ſagacious Sir *Iſaac Newton* had the ſtrong ſcent of ſomething like this which I call æther. What I mean is his

laſt paragraph in his *Principia*, and which, without doubt, he inſerted to excite others to purſue the ſcent which he was forced to leave; his words tranſlated into Engliſh are.

“ I might now add ſomething of a moſt ſubtile ſpirit, which pervades all groſs bodies, and is hidden in them, by the power and action of which the ſmalleſt bodies attract each other, while within the ſmalleſt diſtance, and by the force thereof cohere together; by the force of this ſpirit electric bodies act, both in attracting and repelling ſmall bodies near them; by its power and action light is emitted, reflected, refracted, and inflected, and bodies are heated, and all ſenſation in animals and their voluntary motion is perform'd by the vibration of this ſpirit, continued from the external organs of the ſenſes of the brain to the muſcles, through the ſolid fibres of the nerves. But theſe things cannot be explain'd in a few words, neither am I furniſhed with ſufficient experiments to determine and ſhew the laws, which govern this ſpirit in its actions.”

CHAP. II.

Of the Cauſe of Gravitation.

1. SIR ISAAC NEWTON, with wonderful ſagacity, has diſcover'd that *Gravitation* is an effect of ſome cauſe or agent, which operates in every part of the univerſe of which we have any knowledge; and he has deſcribed the effects, which are commonly attributed to gravitation as a cauſe: But what that cauſe is, whether it acts by attraction or pulſion, he has no where determin'd. And tho' in ſeveral parts of his writings (in the laſt editions) he has more expreſſly declared his opinion, that the agent, which makes all bodies gravitate towards each other, acts in pulſion; yet the method which he has taken to explain this pulſion, has not given that general ſatisfaction which the other parts of his writings have: and he having at firſt explain'd himſelf, that gravitation was caus'd by attraction, his followers have frequently been puzzled, and foreigners have received a prejudice to the whole of that doctrine. Suppoſe that gravitation be by attraction, how can two bodies mutually draw each other without ſomething like ſtrings paſſing between them? but the free motion of any other body between theſe two ſhews, that there can be nothing of that kind. Again, the force of gravitation towards any body, being at every diſtance reciprocally as the ſquares of the diſtances, ſeems to ſhew

that it is the effect of ſome emanation from the attracting body: for the force of any emanation from any body, as a center in ſtrait lines, is at every diſtance reciprocally as the ſquares of the diſtances: which is demonſtrated by Dr. *Gregory* in the 48 prop. of the 1ſt book, and 57 prop. of the 3d book of his aſtronomy. But this no way leſſens the difficulty: for how can we conceive that any emanation from a body can make another move towards it. Theſe difficulties, and ſome others, have prevented the general reception of a doctrine, which evidently agrees with all phænomena. If I can ſhew then, how gravitation is perform'd, ſo as one may be able to form a clear conception of it, conſiſtent with all manner of acting of which we have any knowledge, and founded on the principles before explain'd, I hope to do ſomething that will be acceptable to the curious. But before we proceed, it will be neceſſary to make ſome obſervations on action, not originally in the thing itſelf then acting, but communicated to it from ſome other agent.

2. It is neceſſary to diſtinguiſh carefully between the power itſelf of any ſpecies of matter, and the degree of force or of action, or the manner of exerting that power. Motion for example, may be divided from the higheſt velocity that can be conceived, to the loweſt or ſloweſt motion, into an infinite number of degrees: and all theſe different degrees of velocity are different manners of acting in the moving power: the moving power may commu-

nicate different degrees of action or of velocity, as we ſee every day done; but it cannot communicate its power of moving: for that would be to tranſubſtantiate or alter the eſſential differences of things. And the only effects or alterations, which any power or agent can produce on another, are to communicate to this power ſome degree of its action or manner of acting.

3. The degree of action communicated is always in the ratio of the acting to the receiving power. Thus if any quantity of the moving power communicate any degree of motion to a cubic inch of the reſiſting power, or to the æther, it will communicate but half that degree to two cubic inches. This is proved from daily obſervation.

4. There is ſtill a further diſtinction to be obſerved between the effects of the power itſelf, and the effects of the action communicated to another thing. For example, any quantity of æther, having any degree of motion or of reſiſtance communicated to it, cannot communicate a greater degree than what it received. Suppoſe it had received a velocity at the rate of ten miles in a minute, and it communicated a velocity at the rate of ſix miles in a minute to any quantity of reſiſting matter, it could not communicate a velocity equal to twelve miles in a minute to half that quantity. It is otherwiſe as to the power itſelf, for if it communicate any degree however great to any quantity of matter, it will communicate twice, that degree of action to half that quantity. For, when a power is eſſential to a thing, there can be nothing in that thing to reſtrain it.

5. Suppoſe a line of elaſtic matter, or of æther, conſiſting of the points or parts $\underset{\cdot}{a}\ \ \underset{\cdot}{b}\ \ \underset{\cdot}{c}\ \ \underset{\cdot}{d}\ \ \underset{\cdot}{e}\ \ \underset{\cdot}{f}\ \ \underset{\cdot}{g}$ and that any action be communicated to *a*, *a* will communicate the whole force or degree of action it received to *b*, and *b* to *c*, and ſo on thro' the whole line to *g*, to every particle the ſame quantity or degree of action, and this in the ſame moment or inſtant of time to the extremity of the line: becauſe there is no diſtance between the particles of æther.* The reaction of the action communicated to each particle of æther muſt likewiſe be equal; and ſince the reaction of each particle is likewiſe equal in all directions, the reaction of each particle, in a direction oppoſite or contrary to that of the two contiguous particles on each ſide of it, muſt mutually deſtroy or annull each others actions;* conſequently if the action impreſs'd on the firſt particle in this line, or on *a*, be that of the reſiſting power, the elaſticity of the æther will be equally leſſen'd through the whole line, or it will be as much leſſen'd in the particle *g*, or in any intermediate particle, as in the particle *a*. If then the action of motion be communicated to this line, and any body be placed in it, or move in it, the reaction of this motion will be equal in every part of this line: conſequently, if this body be at reſt, it cannot receive any motion by the reaction of this line of æther; or if it were moving, the direction of its

not so all
motion is necessarily
propagate in time -

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* Chap. I. ſect. 4. par. 4.

its motion cannot be alter'd by the reaction of the particles of æther.

6. Suppoſe a triangular ſurface of æther A B C D (fig. 1.) in which the number of particles in each rank increaſe equally; let it be by 2, then whatever degree of action be communicated by any power to the ſingle particle at A, it will communicate $\frac{1}{3}$ to each of the three particles in the firſt rank, and $\frac{1}{5}$ to each of the five particles in the ſecond rank, and ſo on. The ſingle particle at the point communicates the whole force or degree of action which it received, becauſe the æther has no reſiſtance to leſſen it, and it cannot communicate more than $\frac{1}{3}$ to each of the three particles in the next rank, otherwiſe it would communicate more than it has receiv'd. Then the force or degree of action in this caſe communicated to the particles in the ſeveral ranks will be as $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7},$ *&c.* Now $1 : \frac{1}{3} :: 3 : 1, \frac{1}{3} : \frac{1}{5} :: 5 : 3, \frac{1}{5} : \frac{1}{7} :: 7 : 5,$ *&c.* Therefore, the force communicated, continually decreaſes, in a ratio reciprocal to a ſimilar increaſing arithmetical progreſſion; and integers expreſſing the ſame ratio may be ſubſtituted in their place. Thus the integers 9, 7, 5, 3, 1, expreſs the ſame ratios as $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \frac{1}{9}$. If then the action impreſs'd at the point A of the triangular ſurface of æther be that of the reſiſting power, the degree by which the elaſticity of the æther is thereby leſſen'd in the ſeveral ranks, may be well expreſs'd by theſe numbers 9, 7, 5, 3, 1, that is, the elaſticity of the æther will

not true in
elastic Bodies

reverse to 9 7.
what the author
says

will be the more leſſen'd, the nearer they are to the point, and in the preſent caſe in the ratio of theſe numbers. It is evident then that the moment that the action of the reſiſting power is thus impreſs'd on a triangular ſurface of æther, its elaſticity is continually greater as is its diſtance from the point A, and that the elaſticity increaſes equally with its diſtance by one common difference, in the preſent caſe = 2. Since the action of the reſiſting power is communicated in the ſame inſtant from the point to the fartheſt diſtance, and in the next inſtant carried back again from the greateſt diſtance to the point by the reaction of the elaſtic æther, the effects of the action and reaction in the ſeveral ranks of æther, will be as follow, viz. The difference of the ratio of elaſticity between each rank being = 2. The particle at the greateſt diſtance by its greater reaction will not only deſtroy the reaction of the next rank nearer to the point in the direction from the point, but likewiſe ſuperadd a reaction to the particle in the rank nearer to it in the direction towards the point in a ratio, whoſe difference is = 2, and thereby the ratio of the degree or force of reſiſtance, communicated to the nearer point, will become 3 + 1 = 4, 3 being the ratio of the communicated action by which this next particle reacts, and 1 added to it which it received by the reaction of the fartheſt diſtant point. The next neareſt particle to the point will not only have its reaction in the direction from the point deſtroyed, but will have its reaction in the

direction

action and reaction are opposite and equal —
and where reaction is, there must neceſsarily
be reſistance ∴ therefore aether must have
a reſisting power.

direction towards the point increaſed by the reaction of the contiguous particles and remoter from the point A, whereby the degree of reaction in the particles of the third rank from the extremity becomes 4 + 5 = 9, the particles of the fourth rank 9 + 7 = 16, in the fifth rank from the extremity 16 + 9 = 25, *&c.* So that the degree of reſiſtance communicated to the ſeveral particles in the line or direction A C, by the action of the reſiſting power at A, and the reaction of the æther, will be 25, 16, 9, 4, 1. That is, in a ratio reciprocal to the ſquares of the diſtances from the point where the reſiſting power communicated its action; and the differences of the degree of reſiſtance between the ſeveral points in the direction A C, will be reciprocally as their diſtances from the reſiſting power which impreſſed or communicated its action to the æther. Since the action of the reſiſting power is negative to the action of the other powers, the elaſticity of the æther will be every where leſſen'd in proportion to the degree of reſiſtance communicated to its ſeveral parts. If then motion be communicated to this triangular ſurface of æther, and any body be placed therein, or if any thing move in it, that body will, by the reaction o ſmotion in ſeveral parts of the æther, be impelled or ſeem to be attracted towards the point A, or the direction of the moving thing will be changed towards that point, and the velocity with which the body will be impelled, or the force by which the direction of the moving thing is altered, will be in a ratio reciprocal to the diſtance from the reſiſting matter at A, that is, every where equal to the difference of elaſticity of the æther on the ſide remoteſt from the point A, and on the ſide neareſt to it.

7. It is to be obſerved, that equal degrees of action being communicated to the particles which are at equal diſtances from the point A, the reaction of theſe particles in the direction perpendicular to the direction towards the point A, muſt deſtroy each other by the oppoſite reactions being every where equal, and the differences between the degree of action communicated to the ſeveral particles in rays from A, will be greater than the differences between the particles in directions or lines oblique to the rays or directions tending to the point A, and therefore any thing impelled by the reaction of the æther in this caſe muſt tend to the point A. It is likewiſe to be obſerved, that if the action of motion be communicated to æther every where free from the action of the reſiſting power, the reaction of the parts of the æther will be equal in all directions, becauſe there is nothing to leſſen the elaſticity of the one more than the other. But the reaction in the triangular ſurface will be made in the following manner, ſuppoſing that the action communicated to the point be that of the moving power, and the degree of motion communicated to each particle in any one line proceeding from A to be 9, 7, 5, 3, 1; then in the reaction the particle 9 muſt

deſtroy the reaction of the particle 7 towards the point, and add the difference between 9 and 7 to the reaction of 7 in the direction from the point, and thereby the reaction of 7 from the point muſt become = 9. Again, the reaction of 7 now increaſed to 9 from the point, will deſtroy the reaction of 5 towards the point, and add 4 to it in the direction from the point, whereby its reaction in the direction from the point will likewiſe become = 9; and ſo in like manner the reaction of the following particles in the direction from the point will all be = 9, and all of them loſe any reaction towards the point, except the firſt particle which received the motion, and which being equal to the action it received, muſt by its reacton ſtop the action of the moving power during the time of its reaction. This will be farther illuſtrated when we come to conſider the progreſſive motion of light.

8. In the inſtance here before given, I have ſuppoſed the number of particles in each rank, from A, to increaſe equally by 2, that it may more eaſily appear that their force of reaction is reciprocal to the ſquare of their diſtances: but the concluſion will be the ſame if the number of particles or points be ſuppoſed to increaſe equally by 1, or 3, or 4: for in any of theſe caſes, the force by which any thing will be repelled or appear to be attracted towards A, will be every where reciprocal to its diſtance from A: which will eaſily appear by purſuing the preceding method of calculation.

9. Suppoſe in the third place a ſquare pyramid of æther A, B, C, D, E, and that it be divided tranverſly from the point, into any number of ſurfaces, parallel to the ſurface A, B, C, D, as of 1, 2, 3, 4, 5, 6, *&c.* (fig. 2.) and ſuppoſing that the point is of one particle or point, the ſide of the firſt ſurface be of two particles or points, the ſide of the ſecond of 3, the ſide of the third of 4, and ſo on; then the contents of theſe ſurfaces will be equal to 1 (the point) 4, 9, 16, 25, 36, *&c.* If any action be communicated to the ſingle particle or point A, it will communicate to each particle or point of the firſt ſurface $\frac{1}{4}$ of the force of that action, to the ſecond ſurface $\frac{1}{9}$, to the third $\frac{1}{16}$, and ſo on. Then by the force of reaction of each particle, the reaction of each particle in the laſt or largeſt ſurface will be * as 1, of each particle in the next ſurface nearer to the ſummit as $4 + 1 = 5$, of each in the next neareſt as $9 + 5 = 14$, and of each of the next $16 + 14 = 30$, and ſo on: The force of reaction of every point in the reſpective ſurfaces, from A, to the ſurface B, C, D, E, will be as the numbers 55, 30, 14, 5, 1, including the point; whoſe differences are 2, 5, 16, 9, 4, 1, reciprocal to the ſquares of their reſpective diſtances from A. Then, if the action communicated be that of reſiſting matter, the elaſtic force of the æther, by which the particle in each ſurface reflects or reacts, the action of motion will be leſſened in the ratio of theſe numbers 55, 30, 14, 5, 1; and, if motion be communicated to it, the differences of the

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* Par. 5.

the force of reaction of motion of each particle, in different ſurfaces, will be in a ratio reciprocal to the ſquares of their diſtances from the ſummit or point A. If then any thing be placed in this pyramid of æther, this thing will every where be repelled or appear to be attracted towards the point A, in a ratio reciprocal to the ſquare of its diſtance from A. I have here ſuppoſed a quadrangular pyramid, becauſe in it the ratio of the ſurfaces in the ſeveral ſections is the moſt eaſily perceived: but the ſame ratio of the ſeveral ſurfaces, and reaſoning thereon hold, tho' the pyramid were of any number of ſides or angles, or were a cone.

10. A line, a triangular ſurface, and a pyramid of elaſtic matter have only been ſuppoſed, to lead the mind more eaſily, ſtep by ſtep, to conceive what muſt be the effects of the communication of any action to the æther, and of the reaction of that action by the æther: but ſince it was ſuppoſed * that æther fills every ſpace, not occupied by reſiſting matter, and ſo conſequently permeates all the interſtices between the parts or particles, which compoſe bodies of inert or reſiſting matter, it may be juſtly queſtion'd, whether a line, a ſurface, or a pyramid, or any figure or quantity of æther, can any where exiſt ſeparated from the univerſal æther. I ſhall therefore next ſhew, what are the effects of any action communicated to the æther, without ſuppoſing it of any ſhape or figure, or limited and bounded.

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* Chap. I. ſect. 4. par. 15.

11. Suppoſe then a quantity of reſiſting matter placed any where in the univerſal ſphere of æther, ſince this matter or power exerts its action in all directions equally, it may be properly conſidered as a center, from which its action is communicated to the ſurrounding æther, in rays proceeding from it in all directions. In order therefore to find its effects on the æther at all diſtances, let the ſurrounding æther be ſuppoſed to be divided into equidiſtant ſpherical ſurfaces, included within each other, and that the reſiſting power is the common center of all of them. Now geometers have demonſtrated, that ſpherical ſurfaces are to each other, as the ſquares of their reſpective diameters, or as the ſquares of their ſemidiameters, or as the ſquares of their diſtance from their center. Then by a like reaſoning, as in paragraph 9, if the reſiſting power communicate to each particle, in the firſt or neareſt ſpherical ſurface to the center, a degree of action or force as 1, it will communicate to each particle in the ſecond ſpherical ſurface, a degree of force or action as $\frac{1}{4}$, to each particle in the third ſurface a degree as $\frac{1}{9}$, and to the ſucceeding ſurfaces it will communicate to each particle in the reſpective ſurfaces as $\frac{1}{16}$, $\frac{1}{25}$, $\frac{1}{36}$, $\frac{1}{49}$, $\frac{1}{64}$, $\frac{1}{81}$, $\frac{1}{100}$, *&c.* that is, the degree of action communicated to each particle of æther, will be reciprocal to the ſquare of its diſtance from the center A, (fig. 3.) where the reſiſting matter is placed. Suppoſe then the ſphere to which the action of the reſiſting

ſiſting matter at A extends, to be divided into ten equidiſtant ſpherical ſurfaces, the ratio of force communicated to each particle in theſe ſpherical ſurfaces reſpectively, will be as theſe numbers, 100, 81, 64, 49, 36, 25, 16, 9, 4, 1. Then the force or degree of reaction of each particle in the outermoſt ſpherical ſurface will be as 1, of each particle in the ſurface next nearer the center will be as 4 + 1 = 5, of each particle in the next nearer ſurfaces ſucceſſively will be as 9 + 5 = 14, 16 + 14 = 30, 25 + 30 = 55, 36 + 55 = 91, 49 + 91 = 140, 64 + 140 = 204, 81 + 204 = 285, 100 + 285 = 385. That is, the quantity of reſiſting force in each ſpherical ſurface, ariſing from the reaction of each particle of æther, will be, from the center outwards, in the ratio of the following numbers, viz. 385, 285, 204, 140, 91, 55, 30, 14, 5, 1. Now ſince by the nature of the action of reſiſtance, the power of the particles of æther, to react or reflect motion, will be leſſen'd in the ſame ratio every where, according to their diſtance from the center A, where the reſiſting power is placed; therefore if any thing be placed in this ſphere of æther, and motion be communicated to it, that thing will be repelled, or ſeem to be attracted towards that center: becauſe the motion is every where more ſtrongly reflected by the æther on the ſide that is at the greateſt diſtance from the center, and is more weakly reflected on the ſide that is neareſt the center. And the difference between the force by which motion is reflected, on the moſt diſtant ſide of this thing from the center, and its neareſt ſide, is the force by which that thing will be repelled, or ſeem to be attracted towards the center. But the difference of theſe numbers in the ratio which the force of reaction of motion is leſſen'd, are every where reciprocal to the ſquares of the diſtances from the center; and in the caſe ſuppoſed, are as the numbers 100, 81, 64, 49, 36, 25, 16, 9, 4, 1. It is of no conſequence into what number of ſpherical ſurfaces the ſphere of æther be divided, provided they be every where equidiſtant, the ratio in which any body or thing is repelled, will always be reciprocal to the ſquare of the diſtances. For, we do not here treat of the abſolute quantity or degree of force in the ſeveral diſtances from the center.

12. If any thing without motion be placed in the ſphere of æther, to which the action of the reſiſting power is communicated, this thing can have no motion either towards the reſiſting matter, or from it. For ſince the action of reſiſting is negative to motion, and the æther of itſelf has no motion, and it is not ſuppoſed that motion is communicated to it otherwiſe, this thing can acquire no motion, but muſt remain in whatever place of the ſphere it ſhall happen to be placed. But if that thing move, or motion be communicated to the ſphere of æther, then it will be repelled towards the reſiſting matter in the center of the ſphere, in the manner deſcribed in the preceding paragraph. When we conſider that light from the

ſun and ſtars and planets and reflected light, is found every where, and that all the planets are in motion, no part of the æther can at any time be without motion communicated to it, and no time or place can be found, wherein gravitation may not be perceived. It is to be remember'd, that the reaction of reſiſtance and of motion are both in the ſame inſtant; for the æther, as before obſerv'd, is by turns in a ſtate of receiving any action, and of reacting any action.

13. It follows then, that no ſpecies of matter has in itſelf a power of attracting any other, or gravitating towards any other; but that this apparent attraction or gravitation is truly and really perform'd by pulſion, or more properly is the effect of the joint actions of the moving, reſiſting, and elaſtic powers. However, as the Copernican aſtronomers often ſpeak of the motion of the ſun and ſtars, tho' they knew that they are ever at reſt, ſo I think it may be allowed to ſpeak of attraction and gravitation, and to conſider this apparent action as if it were real. When attraction is conſidered as a real action in place of the degree of action of the reſiſting power communicated to the ſeveral ſurfaces, the differences of thoſe degrees of actions between the adjoining ſurfaces is to be taken; and then the method of calculating will be the ſame as if a real attraction exiſted.

A B C D E

F G H.

14. If a line conſiſting of any number of particles of the reſiſting power be taken, ſuppoſe it conſiſt of eight particles as in this margin, the middle of that line is between D and E, then the action of the particles D and E in the directions D H, and E A muſt be deſtroyed or render'd ineffectual, by their equal and contrary actions, and conſequently will be o. But the action of D towards A being equal to the difference of D E F G H and A B C will be $5 - 3 = 2$, that of C will be $= 6 - 2 = 4$, that of $B = 7 - 1 = 6$ and $A = 8 - 0$. Then the action of the reſiſting matter, in a line of ſuch matter will be exerted towards its extremities, from the middle both ways; and the force of action in every particle will be in the ratio of its diſtance from the middle, or will be in the ratio of arithmetical proportionals, increaſing both ways from the middle. The particles of reſiſting matter being ſuppoſed to be at equal diſtances, the action by one is equally communicated to the next, by the æther between them. Then the reaction of the æther ſurrounding A will be 8, of B $8 + 6 = 14$, of C $14 + 4 = 18$, and D $18 + 2 = 20$. The force of reſiſtance, communicated by the reaction of the æther in this line, will increaſe to its middle; and the differences of force in each particle will increaſe in the ratio of the diſtance from the middle, or in the ratio of increaſing arithmetical proportionals from the middle. If then mo-

tion be communicated to this line of æther, any thing placed in it muſt be repelled towards its center, and the force, by which it will be repelled, or ſeem to be attracted, will be every where in the ratio of the diſtance from the middle.

15. It may be obſerved from the 11th paragraph and 3d figure, that, if lines be ſuppoſed to be drawn from every point of the outermoſt ſurface of the ſphere of æther, ſurrounding a quantity of reſiſting matter, to the oppoſite point of that ſphere, the whole quantity of reſiſting action, which is or can be communicated to each particle of æther compoſing thoſe lines, is communicated to them, and likewiſe the whole of the action communicated is reflected in the ſame line but oppoſite direction by the reaction of the æther. It was obſerved likewiſe, that no part of the reaction of the adjoining particles of æther can be communicated to the particles of that line in the direction of that line. Now, then ſuppoſe a ſpherical body of reſiſting matter, and that lines are drawn through its center from every point of its ſurface to the oppoſite point, or that it is compoſed of an infinite number of lines croſſing in the center, the quantity of reſiſting action communicated to the æther which is contain'd in the interſtices of that ſpherical body, and the reaction of the æther, in lines paſſing through the center, will be as in the preceding paragraph: and the force of repulſion, or apparent attraction towards the center of the ſphere, when motion is communicated to the æther contain'd

tain'd within it, will be every where as the diſtance from the center.

The ſame may likewiſe be demonſtrated or ſhewn in the following manner. Of all the lines, paſſing through any point or particle of a ſphere, thoſe are the longeſt which paſs through the center; therefore theſe lines have the moſt reſiſting matter in them, or more than any other line not paſſing through the center: conſequently, the elaſticity of the æther in thoſe lines muſt be more leſſen'd than in any other; and therefore the reaction of the æther muſt be in lines paſſing through the center. Likewiſe the length of every line within the ſphere paſſing through any particle or point of the ſphere and the center, is longer on that ſide of the particle on which the center is, than on the oppoſite ſide towards the ſurface, and conſequently the elaſticity is moſt leſſen'd on that ſide, and the reaction of the æther from that point muſt be towards the center. Again, ſince every line of a ſphere paſſing through the center, is equally divided at the center, the oppoſite directions of reaction in every line muſt be equal at the center, and therefore the force of repulſion or apparent attraction at the center muſt be $= 0$. And ſince the difference of the lengths of the lines paſſing through every particle and the center, on the one ſide and oppoſite ſide of that particle, increaſe as the diſtance of the particle from the center; the difference of the degrees of reaction on each ſide of every particle muſt be as its diſtance from the center. The like kind of reaſoning applied to the ſphere of æther ſurrounding the reſiſting body, will ſhew that the repulſion or apparent attraction muſt be towards the reſiſting body, conſidered as a point and center of the ſphere.

16. Then if any ſpherical body be placed any where in an unlimited quantity of æther, and the æther be ſuppoſed to be divided as well within the body as without it, into any number of equidiſtant ſpherical ſurfaces, concentric with the ſpherical body, the elaſtic force of the æther will be leaſt at the center, and will continually encreaſe from the center; but in different ratios within the body and without it. Within the body it increaſes as the ſquares of the diſtances from the center, and the differences between each ſpherical ſurface in this diviſion, or the ratio of the force of repulſion, or apparent attraction, is directly as the diſtances; but without the ſpherical body, the elaſtic force increaſes in the ratio of numbers, whoſe differences are reciprocal to the ſquares of the diſtance from the ſurface of the ſpherical body, and conſequently the difference of elaſticity between each two particles, is reciprocal to the ſquares of their diſtance from that ſurface, and the force of repulſion or apparent attraction is in the ſame ratio. For example, putting the circle B (fig. 3) for the ſurface of a ſpherical body, divided into four concentric equidiſtant ſpherical ſurfaces within the body (reckoning the exterior ſurface as one) and ſix without

it, the ratio in which the action of a reſiſting body is communicated to the ſphere of its action from the center of the body outwards, will be in the ratio of the following numbers, viz. 145, 136, 118, 91, 55, 30, 14, 5, 1, or the elaſticity of the æther will be leſſen'd in proportion of the following numbers, ſuppoſe its greateſt elaſticity without the ſphere of action of the reſiſting body be 1000, viz. 999, 995, 986, 970, 945, 909, 882, 864, 855, and the ratio of gravitation will be as the differences of thoſe numbers, viz. 1, 4, 9, 16, 25, 36, 27, 18, 9, 0.

17. Gravitation towards the center of any ſpherical body may be repreſented by two different decreaſing progreſſions, the greateſt term common to both being at the ſurface of that body: one of which progreſſions decreaſes towards the center, reciprocally to the diſtance from the ſurface, the other decreaſes outwardly from the ſurface reciprocally to the ſquares of the diſtance, and the gravitation is greateſt at the point of contact. Let C (fig. 4.) be the center of any ſpherical body, S its ſurface, and T the outmoſt limit to which the action of its reſiſtance, or its apparent attraction, extends; divide C S equally into two, and S T likewiſe at A and A, then any thing will deſcend from S to A in half the time, by the force of gravitation, that it will from A to C; and any body will, by the force of gravitation, deſcend in a quarter part of the time from A to S, that it will from T to A. If C S be divided into three equal parts at B and D, any thing

will deſcend from S to D in one third part of the time, and from D to B in one half of the time, that the ſame thing will deſcend from B to C: and if S T be in like manner divided into three equal parts at B and D, any thing will deſcend from D to B in a quarter part of the time, and from B to S in a ninth part of the time, that the ſame thing by the force of gravitation deſcends from T to D.

18. Into whatever number of parts the diſtance to which the action of reſiſting matter, and conſequently the apparent attraction, extends, be divided, the ratio of the force of gravitation in each of thoſe parts may be found, and if the diſtance be divided infinitely, the degrees of gravitation will become an infinite ſeries. But an infinite ſeries of proportionals decreaſing or increaſing as the diſtances, or as the ſquares of the diſtances, may be in any finite diſtance. If the ratios of the difference between any finite part of ſuch infinite ſeries be finite, the diſtance to which the whole ſeries extends muſt be finite. Since then the force of gravitation is not found to be infinitely great at any finite diſtance, it cannot be true that all bodies mutually attract each other in an abſolute and unlimited ſenſe.

19. The numbers 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, are the ſquares of numbers increaſing in their natural order 1, 2, 3, *&c.* to 10: and theſe ſquares are formed by a continual addition of another arithmetical progreſſion,

whoſe common difference is the double of the firſt term: viz. they are formed by the continual addition of 1, 3, 5, 7,, 9, 11, 13, 15, 17, 19. So likewiſe 4, 16, 36, 64, 100, are the ſquares of an arithmetical progreſſion, and are compoſed by continual addition of this other arithmetical progreſſion, viz. 4, 12, 20, 28, 36, whoſe common difference is 8. The following numbers in like manner, viz. 0, 25, 1, 2.25, 4, 6.25, 9. 12.25, 16, *&c.* to 100, are ſquares of an arithmetical progreſſion, and are form'd of the following arithmetical progreſſion, viz. 0. 25, 0.75, 1.25, 1.75, 2.25, *&c.* to 9.75, whoſe common difference is 0.5. Then taking any number for the greateſt term of the ſquares, any number of ſquares to arithmetical proportionals may be found decreaſing to nought. And likewiſe means to any number of ſquares of an arithmetical progreſſion may be found.

Call the given greateſt term — —— m
The number of terms deſired — n
The common difference of the arithmetical progreſſion which form the ſquare } d
The firſt term — — — a
The laſt term of that arithmetical progreſſion l

Then $\frac{m}{nn}=a$, $2a=d$, $\frac{2m}{n}=l$.

And the arithmetical progreſſion forming the ſquares will be a, $3a$, $5a$, $7a$, *&c.*

This gives an eaſy method of forming equations for the orbits of the planets, the greateſt equation or excentricity being given, as will be afterwards more fully ſhewn when we come to treat of that matter.

20. If the velocity (or the time being reciprocal to the velocity) with which any body paſſes by gravitation through three equal and continued diſtances, be given, the diſtance from that place to the limit of attraction, or to the extent of action of the reſiſting body (which is ſuppoſed to attract) on the æther, may be found thus: Call the greateſt velocity found a, the middle velocity b, and the laſt c, $a - b = d$, and $b - c = e$, and $d - e = f$, then $\frac{d + \frac{1}{2}f}{f} = z$, or $\frac{e + \frac{1}{2}f}{f} = z$, the number of ſpaces, each equal to the ſpace between a and b, or between e and c, which thoſe ſpaces are from the limit or extent of attraction. For example, let $a = 100$, $b = 81$, and $c = 64$, then $d = 19$, $e = 17$, and $f = 2$, therefore $\frac{19 + 1}{2} = 10 = z$, or $\frac{17 + 1}{2} = 9 = z$. Again, let $a = 100$, $b = 64$, $c = 36$, then $d = 36$, $e = 28$, and $f = 8$, and $\frac{36 + 4}{8} = 5 = z$. Or let $a = 100$, $b = 90.25$, $c = 81$, then $d = 9 \cdot 75$, $e = 9 \cdot 25$, and $f = 0 \cdot 5$, and $\frac{9 \cdot 75 + 0 \cdot 25}{0 \cdot 5} = 20 = z$, or $\frac{9 \cdot 25 + 0 \cdot 25}{0 \cdot 5} = 19 = z$. Here it may be obſerved, that the ſmaller theſe differences are in proportion to the diſtances a, b, c, the greater the diſtance to the limit of attraction.

21. Lines may be conceived as compoſed of points, ſurfaces of lines, and ſolids of ſurfaces: then a triangular ſurface may be conceived as compoſed of an infinite number of lines increaſing in an arithmetical progreſſion, and a cone of an infinite number of ſurfaces continually increaſing ſo as that their differences are a ſeries of increaſing arithmetical proportionals: therefore inſtead of a ſeries produced by the continual addition of arithmetical proportionals, or of a ſeries produced by the continual addition of the ſquares of arithmetical proportionals, we ſhall hereafter ſay a ſeries of the ſquares of arithmetical proportionals, or a ſeries of their cubes. It is true, that while the differences of the ſeries of the arithmetical proportionals can be expreſſed in finite numbers, the continual addition of them does not make perfect triangular ſurfaces or cones, but as the number of lines or ſurfaces compoſing the triangular ſurface or cone increaſe, they come nearer and nearer to true triangular ſurfaces, and true cones; and if the number be an infinite ſeries, they will be perfect triangular ſurfaces, or perfect cones. If any triangular ſurface, for example, be divided into any ſmall number of lines, theſe lines muſt have ſome breadth, and the oppoſite ſide of the triangular ſurface in which theſe lines terminate will not be ſtrait lines, but like ſo many ſteps of ſtairs, the more of theſe ſteps they muſt be the ſmaller, and the difference of the lines leſs; therefore if the number of lines be infinite, theſe ſteps muſt vaniſh, and the ſides become ſtrait lines. Now if

the parts or particles of the æther be infinitely ſmall (as perhaps they may afterwards be proved to be) there muſt be an infinite number of ætherial ſurfaces in every finite ſpace, and therefore theſe infinite ſeries make or compoſe ſolids in a truly mathematical ſenſe. Again, if they do compoſe ſolids in a truly mathematical ſenſe, the particles of æther muſt be infinitely ſmall. See *Wallis's Arithmetica Infinitorum.*

22. Suppoſe any cubical quantity of reſiſting matter A, B, C, D, E, F, be divided into any number of ſquare ſurfaces as in the figure [fig. 5.] into the ſurfaces 1, 2, 3, 4, then let the force of attraction in the undermoſt ſurface 4, be *x*, the force of attraction of the ſecond ſurface from the bottom, at the upper ſurface will be 4*x*, of the third ſurface 9*x*, and the uppermoſt ſurface 16*x*, becauſe the force of attraction is every where reciprocal to the ſquares of the diſtances. By the 21ſt Propoſition of Dr. *Wallis's Arithmetica Infinitorum*, the ſum of an infinite ſeries of arithmetical decreaſing proportionals is equal to one third of the ſum of as many ſurfaces all equal to the greateſt; but any cubical quantity may be divided into an infinite number of ſurfaces, therefore the force or degree of attraction is one third at the ſurface, of what it would be if they retain'd the ſame degree or force of attraction to the fartheſt diſtant ſurface of that cubical quantity.

23. Since then every quantity of reſiſting matter loſes two thirds of its apparent attraction, in the proportion of its bulk, the greater any body is, the leſs is the force of its

attraction in the proportion of its bulk; and the leſs it is, the greater, in the ſame proportion is the force of attraction; conſequently, if the body be infinitely little, the force of attraction muſt be equal in proportion to the quantity of matter in the infinitely little body: for in this caſe, the difference between the proportion of attraction, and of the quantity of matter, becomes equal to 0.

24. It follows, from the force of attraction being greater in little bodies than in greater, that their attraction likewiſe extends to a greater diſtance; that is, in the proportion of the quantity of reſiſting matter in each. This may likewiſe be proved thus; the extent of any infinite ſeries, decreaſing as the cubes or ſquares of arithmetical proportionals, is as the ſquare root or cube root of the firſt term of each; and this holds true likewiſe in finite ſeries, if the diſtance between the terms of the ſeveral ſeries be equal. For example; ſuppoſe two quantities of matter, the one as 100, the other as 300, their force of attraction at the ſurface is as 33 $\frac{1}{3}$ to 100, and the extent of their attraction nearly as 6 to 10. Again, if the quantity of matter be as 300 to 3000, the force of attraction will be as 100 to 1000, and the extent of their attraction as 10 to ſomething leſs than 32.

25. Tho' the apparent action of attracting and gravitating really ariſe from the ſame cauſe, yet the force of gravitation, in different bodies, may be different from the force of attraction in the ſame bodies; that is, ſuch bo-

dies as are heavieſt may not have the greateſt force of attraction. For gravitation is cauſed by the action of the æther on every particle of matter, of whatever kind it be, whether reſiſting or moving, that is contained in the gravitating body; and tho' there were different ſpecies of reſiſting matter, one endowed with a greater power of reſiſtance than the other, the action of the æther may be equal on all of them. Here I do not conſider the effects of any motion, in a direction oppoſite to gravitation, or combining with it, as the throwing a ſtone up into the air is not ſaid to leſſen the natural or intrinſic weight thereof, or throwing it down to the earth to increaſe its weight: But the force of attraction is only as the quantity of reſiſting matter in thoſe bodies; and if there be different ſpecies of reſiſting matter, the ſame quantity may have a different degree of attraction. By obſerving therefore the differences between attraction and gravitation, in the ſame bodies, and by contriving proper experiments for that purpoſe, or perhaps by obſerving the phænomena of the great heavenly bodies, the quantity of reſiſting matter and moving matter, or the different ſpecies of reſiſting matter, may be diſcovered. I ſuppoſe, that the æther freely permeates between and round all the parts or particles of any body, and acts upon every one of them. It is beſide my preſent purpoſe to prove this, but perhaps it may be afterwards done, when it can be ſhewn by what power the parts of bodies cohere.

26. If two equal ſpherical bodies be placed within the diſtance to which the action of the reſiſting matter in each extends, and conſequently within each other's ſpheres of apparent attraction, the æther will have its elaſticity leſſened by the action of both, every where in a ratio reciprocal to the cubes of the diſtance from each. Let there be two quantities of reſiſting matter, whoſe force or degree of action on the contiguous ſurface of æther in each, is equal to 1240, and that theſe two bodies are placed at the extremity of each other's ſphere of action: then the æther will have its elaſticity leſſened by each, in the ratio of the following two firſt lines of numbers, and by their joint action in the ratio of the third line of numbers,

1240, 1015, 819, 650, 506, 385, 285, 204, 140, 91, 55, 30, 14, 5, 1
1, 5, 14, 30, 55, 91, 140, 204, 285, 385, 506, 650, 819, 1015, 1240

1241, 1020, 833, 680, 561, 476, 425, 408, 425, 476, 561, 680, 833, 1020, 1241
221, 187, 153, 119, 85, 51, 17, 17, 51, 85, 119, 153, 187, 221
34, 34, 34, 34, 34, 34, 0, 34, 34, 34, 34, 34, 34

The fourth line of numbers ſhews the difference between the terms of their joint action, and the fifth line ſhews that the numbers in the fourth line are arithmetical proportionals, whoſe common difference is 34. From theſe numbers it appears, that the ſum of two equal ſeries of the cubes of arithmetical proportionals, placed the contrary way, form two ſeries of the ſquares of arithmetical proportionals, the leaſt term of each of which begins in the middle of the two equal oppoſite ſeries;

or more generally where two terms of the oppoſite ſeries are equal, as in the inſtance before us, at the 8th term in each, and where in both ſeries that term is 204. Here it is evident likewiſe, that the elaſticity of the æther is not ſo much leſſen'd at this term, as it is at any other term or diſtance between the two quantities of reſiſting matter; and laſtly, that the ratio of the increaſing of the action of the two reſiſting quantities on the æther, are as the ſquares of the diſtance from the middle point towards either of the reſiſting bodies or quantities; or reciprocally as the ſquares of the diſtance from either body towards this middle point. Conſequently, ſince the force of apparent attraction is every where as the difference of the action of the reſiſting matter, in the ſeveral terms or diſtances, the force of attraction towards either of theſe bodies will be directly as the diſtance from this middle point, or reciprocally as the diſtance from either body towards the middle point.

27. The greateſt force of the elaſticity of the æther, leſſened through the whole diſtance between two equal bodies of reſiſting matter, by their joint action, is in the middle point between them: For the ſum of each two terms of two oppoſitely decreaſe progreſſions as the cubes of their diſtance, continually decreaſing to that point, and the ſum of each two terms in this caſe, is the ratio in which the elaſticity of the æther is leſſened at the reſpective diſtance. A third ſmaller body therefore, placed any

where between this middle point and either of the two greater bodies, muſt be repelled, or gravitate towards that greater body to which it is neareſt. But a ſmall body placed in the middle between two large equal bodies, will remain at reſt.

28. This middle point between equal bodies is called the *limit* of their ſeveral attractions. The limit between unequal bodies muſt be always nearer the leaſt; and to find the limit between any two unequal bodies, firſt find the diſtance from the greaterbody at which the force of attraction is equal to the abſolute force of the ſmaller body, then if the ſmaller body be placed at that diſtance, or nearer to the greater body, there is no attraction to the ſmaller body in the line between it and the greater body; but if it be placed farther from the greater body, then the middle between the ſmaller body and this diſtance is the limit of attraction between the two unequal bodies. For, from the place where the force of attraction of the greater body is equal to the abſolute force of the ſmaller, the elaſticity of the æther, between that place and the ſmaller body, is leſſened in the ſame manner as between equal bodies. By ſuppoſing one of the terms, of one of the two progreſſions in paragraph 26, to be the abſolute degree of action of any quantity of reſiſting matter, and that quantity either to ſtand at the ratio of diſtance from the other quantity it does in the example, to be removed nearer to, or farther from that other quantity, the ratio of diſtance of theſe ſeveral quan-

tities from the limit of attraction between any of them, and the other quantity, in their ſeveral poſitions, may be eaſily conceived.

29. If two equal bodies A B be ſo placed, (fig. 6) that the ſpheres of their action interſect each other, then the limit of their attractions will be in the middle point C of a line joining the centers of theſe bodies; and if a plane C D, perpendicular to this line, paſs through the middle point C, the limit of the attraction of theſe bodies in any place between them without the line A B, will be in this plane: for A D : B D : : A C : B C; and the limit of attraction will extend from C, till the line A D be equal to the extent of the ſphere of attraction. Then if any ſmall body be placed any where in this circular plane or ſurface C D, it will be repelled towards C, and not towards either of the bodies A or B: becauſe the action of the æther on the ſides towards A and B is equal, but on the ſide next C the elaſticity is more leſſened than on the ſide more diſtant from C; and the force of attraction, in this plane towards C, will be every where reciprocal to the ſines of the angle D B C or D A C, applied to the radius A D or D B, or in the ratio reciprocal to the tangents of the angle D B C or D A C; becauſe the action of the bodies A and B, will be every where as if the bodies were removed from each other to the diſtance A D B, and the force of attraction in the middle place between any two equal bodies, leſſens as their diſtance increaſe. It is evident

likewiſe, that the nearer any two bodies are to each other, the greater will be the ſemidiameter of this circle the limit plane of their attractions be.

30. Let A and B as before, be two equal bodies of reſiſting matter placed at the diſtance A B (fig. 6.) and within each other's ſphere of action; let A F D, be part of a ſpherical ſurface whoſe ſemidiameter is A B, and D B, another part of a ſpherical ſurface of the ſame diameter from the center A; and let FE,FE,&c. be ſpherical ſurfaces having their common center in A, and let ED,ED,&c. be ſpherical ſurfaces having their common center in B; then DC is the ſemidiameter of the limit circle of attraction between A and B. The ratio of attraction in the ſpherical ſurface AFD will be every where the ſame as if the body B were abſent: becauſe when equal quantities are added to different quantities, their differences are not altered thereby, but remain the ſame after ſuch addition as before. For a like reaſon the force of attraction towards B in the ſeveral ſpherical ſurfaces E F and E D, having A for their common center, will every where be from E to F reciprocal to the ſquares of the diſtances from B; conſequently, the attraction in thoſe ſurfaces towards A, increaſe as the ſquares of the diſtances from B. If the ſpherical ſurface croſs the limit plane, the attraction will increaſe both ways from D, towards A on the one ſide, and towards B on the other. If a little body be placed any where within the ſpherical ſurface AD, BD, and not in the limit plane between, it will be attracted towards

either A or B, according to which ſide it ſhall be on; but it will not move in a ſtrait line, but in a curve line, convex towards the line joining the centers of the bodies A and B; and the nearer that body is to the limit plane, the greater will the curvature be: becauſe the elaſticity of the æther is every where more leſſened on the ſide of the little body next to the line A B, than on the oppoſite ſide; and the ratio of this difference, in proportion to the attraction towards A, is greater, the nearer the body is to the limit plane.

31. If two bodies, a greater and a ſmaller, be ſo placed, that the attraction of the greater, at the diſtance at which the ſmaller is placed, be greater than the abſolute force of the ſmaller, then this ſmaller body cannot attract a third body placed any where between them. But in this, to prevent miſtakes, a proper regard muſt be had to the meaſure with which the diſtance is computed. For example, if the meaſure be a ſemidiameter of the earth, and if at the diſtance from the ſun, computed by this meaſure, the force of attraction of the ſun be equal to the abſolute force of the earth, a third body on the ſurface of the earth would not gravitate towards it. But if the meaſure were the earth's diameter, and if at the diſtance from the ſun according to this meaſure, the ſun's attraction be equal to the earth's abſolute force of attraction, then a third body will gravitate towards the earth, any where not more than the earth's ſemidiametar diſtant from its ſurface. The reaſon is evident,

becauſe the difference of the force of the æther on the oppoſite ſides of any two bodies, is as the ſquares of their diameters. If a ſmall body be removed at a great diſtance from a large body, it may attract a third body, which it could not in its firſt ſituation; and the diſtance in this caſe, at which a ſmall body can attract a third body, will increaſe, as its diſtance from the great body increaſes. This may be of uſe in explaining many phænomena among ſmall bodies, where the diſtance between them is continually changing, or is by ſome force or other changed.

32. If two ſpherical bodies be ſo far placed within the ſphere of each other's action, that each extend beyond the other, then the force of gravitation of a third little body to either of them, while in the line paſſing through the center of the bodies, is, as before obſerved, reciprocally as the diſtance from that body to which it gravitates, or directly as the diſtance from the limit of attraction between the two bodies, while it is between the two bodies, but reciprocally to the ſquares of its diſtance while it is on the ſide oppoſite to the other body. For the force of attraction of each of theſe bodies, on the oppoſite ſide to each other, is increaſed in the ſame manner, as if their quantity were increaſed by another quantity of reſiſting matter, equal to the force of the other, at the diſtance at which the other is placed. Thus ſuppoſe the force of the bodies A and B to be 1240, and that they are placed

within each other's ſphere of action, as in the following numbers :

A Limit B

1015,1240,1015,819,650,506,385,285, 204, 140, 91, 55, 30, 14, 5, 1

91, 140, 204,285,385,506,650,819,1015,1240,1015,819,650,506,385,285

1106, 1380, 1219,1104,1035,1012,1035,1104, 1219, 1380, 1106, 874, 680, 520, 390, 286

274, 161, 115, 69, 23, 23, 69, 115, 161, 274, 232, 194, 160, 130, 104

113, 46, 46, 46, 0 46, 46, 46, 113, 42, 38, 34, 30, 26

0 0 0 0 4, 4, 4, 4

Where the firſt line ſhews the force of the action of the body A on the æther, every where reciprocal to the cubes of the diſtances: the ſecond line the force of the body B at the like diſtances from it: the third line the united forces of both bodies at the ſame diſtances : the fourth line ſhews the difference of the united forces between each diſtance, or the force of gravitation: and the fifth line ſhews that theſe differences between the bodies are as the ſquares of arithmetical proportionals. The number 113 under both A and B ſhews the difference of attraction on the ſide next the other body, and the ſide oppoſite to it, or furtheſt from it.

33. It is likewiſe evident from theſe numbers, that the elaſticity of the æther, at the ſeveral diſtances in the line paſſing through the centers of two bodies, placed at a diſtance from each other, but within the ſphere of each other's action, are different from what they are when each are ſingle, or at a greater diſtance than that of the ſpheres of

their action can interſect each other: that if theſe bodies placed within the reach of each other's actions, be removed to a greater diſtance, the elaſticity of the æther increaſes between them, and is leſſened by their approach to each other: that by their approach towards, or removing from each other, the elaſticity at ſeveral diſtances is leſſened or increaſed, in a ratio of the cubes of the diſtance reciprocally or directly. And ſince the difference of oppoſite progreſſions of the cubes of arithmetical progreſſions, are as ſquares of arithmetical progreſſions, and theſe differences are continually leſſened as the greateſt terms of the progreſſion approach the attraction towards either of the two bodies, if they approach each other, leſſens as the ſquares of the diſtance between them: But the attraction towards any of them on the oppoſite ſide, by their approaching to each other, will increaſe reciprocally as the ſquare of the diſtance between them.

34. If the action of the body B extend beyond the body A, (fig. 6.) then, ſince in the line A G, the force of gravitation towards A is increaſed, as if a quantity of reſiſting matter equal to the action of B, at the diſtance A B were added to it, if any third little body move in any circle concentric to A, viz. in E F G, the force of gravitation towards A, from E to F, and from F to G, will increaſe as the ſquares of the diſtances from B. For the gravitation to A is every where leſſened reciprocally to the ſquares of the diſtances from B, while within the

ſpherical ſurfaces A D, B D, and is no way affected by it in the ſpherical ſurface A F D, but increaſed by the attraction of B in the line A G, as was before obſerved. If the circle in which the third little body move croſs the limit plane between A and B, the gravitation will continually decreaſe from E to D, as the ſquares of the diſtances from B, and afterwards increaſe in the ſame ratio, but the gravitation on the ſide of the limit plane next to B will be towards B, and not towards A. So likewiſe, if the little body move in any circle H G, concentric to the body B, its gravitation will continually increaſe towards G, in a ratio reciprocal to its diſtance from A.

35. Suppoſe A a ſmaller body, whoſe abſolute force of action is to the abſolute force of another body B, as 204 to 819, and let them be placed at ſuch diſtance of the ſeveral terms of the progreſſion, by which their reſpective force at ſeveral diſtances is expreſſed, as in the following numbers, viz.

							A		L				C		B
			1,	5,	14,	30,	55,	91,	140,	204,	285,	385,	506,	650,	819
1,	5,	14,	30,	55,	91,	140,	204,	140,	91,	55,	30,	14,	5,	1	
1,	5,	14,	31,	60,	105,	170,	259,	231,	231,	259,	315,	399,	511,	651,	819
4,	9,	17,	29,	45,	65,	89,	28,	0,		28,	56,	84,	112,	140,	168
5,	8,	12,	16,	20,	24,	6,	28,			28,	28,	28,	28,	28,	28
3,	4,	4,	4,	4,	37,	33									

Where the firſt line expreſſes the force of B at ſeveral equal diſtances, the ſecond line the force of A at the ſame diſtances from it, the third line the united forces of both at the ſame diſtances, the fourth line expreſſes the differences of theſe terms, or the force of attraction towards the one or the other body, and the fifth line ſhews whether theſe differences be arithmetical proportionals, or the ſquares of arithmetical proportionals; where obſerve, that the number 61 expreſſes the difference between the force of attraction of A, on the ſide next to B, and the oppoſite ſide; and that as the elaſticity of the æther is more leſſened on the ſide towards B of the little body A, than it is on the oppoſite ſide, that the body A is repelled towards B. The ſame would happen to B, if it were placed ſo near that the action of A extended to it; but as the differences of the elaſticity are not ſo great between the two ſides of B, as of A, it will not be repelled with ſo much force towards A, as A is towards it. It likewiſe appears, from conſidering theſe numbers, that the limit of attraction L, between two different quantities, or degrees of force of reſiſting matter, is at a diſtance from each reſpectively in a ratio as the ſquares of the cube root of their reſpective quantities of reſiſting force: and the place where theſe two bodies would meet, commonly called their center of gravity C, will be reciprocally to the ſquare of their reſpective cube roots. If the action of the leaſt body do not extend to the greateſt body, aſſume the degree of action of the greateſt body at the place to which the action of the little body extends, as if it were the abſolute force of the greateſt body, and reckon the reſpective diſtances from thence, in order to find either the limit of their reſpective attractions, or their center of gravity. It is likewiſe eaſily perceived, from theſe numbers, that the celerity or ſpace through which two different quantities of reſiſting matter will paſs, by their mutual attraction before they meet, is reciprocal to the ſquares of the cube roots of the reſpective quantities, or reciprocal to their abſolute power of attraction.

36. I ſhall in the laſt place apply this theory of the effects of the action of reſiſting matter on the action of the æther, to the explaining a moſt remarkable phænomenon which puzzled all the philoſophers till Sir *Iſaac Newton* explained it. Let A be the moon (fig. 7.) whoſe action of reſiſting extends on the æther beyond the body of the earth B, whoſe exterior ſurface is repreſented by the circle C C, and let the numbers on the line A B repreſent the effects of the reſiſting matter in the moon on the æther at the ſeveral diſtances from the moon, or the ratio in which the elaſticity of the æther is leſſened, at the ſeveral diſtances from the moon: then any fluid on the ſurface of the earth A, will be leſs preſſed in the places (and near them) where the ſurface is cut by a line A B, paſſing through the centers of the moon and earth, than where the ſurface is cut by a ſpherical ſurface C B C paſſing through the center of the earth, and every way equidiſtant from the

moon: for the elaſticity of the æther being every where equally leſſened on the ſurface of any ſpherical body, by the reſiſting matter of that body, no difference can be occaſioned by it on any part of the ſurface; but the elaſticity of the æther on the ſurface next the moon, is more leſſened by the reſiſting matter in the moon, than at the diſtance of the earth's center; and more at the center, than at the ſurface fartheſt diſtant from the moon. Now of any three terms of a progreſſion, increaſing as the cubes of a ſeries of arithmetical proportionals, twice the middle term is leſs than the ſum of the other two terms; therefore the elaſticity of the æther will be greater on the two ſides cut by the ſpherical ſurface C C, than on the oppoſite ſides cut by the line A B: and ſince the elaſticity at the ſide fartheſt from the moon, is greater than on the ſide next the moon, the point of equal preſſure between theſe two ſides will not be at the center of the body, but at ſome other point nearer to the ſurface next the moon; ſo that the elaſticity at theſe two oppoſite ſides, ſhall in each be equal to one half of their ſums; but one half of the ſum of the elaſticity of both ſides cut by the line A B is leſs, or the the ratio at which it is leſſened is greater, than the elaſticity at C. Thus, in the ſcheme, 140 is the ratio by which the elaſticity is leſſened at the ſide next the moon, and 14 the ratio in which it is leſſened at the ſide fartheſt from the moon; their ſum is 154, the half of which 77 greater than 55, ſhews that the elaſticity of the æther is more leſſened at theſe two oppoſite ſides, than on the ſides cut by the ſurface C B C. And ſince the elaſticity is greater on the ſides C C, any fluid will be more preſſed there, and conſequently muſt ſwell towards the points cut by the line A B. That this alteration of the center of elaſticity, and conſequently of the center of gravity in the body B, may be better underſtood, it is to be obſerved, that it was before ſhewn * that the elaſticity of the æther in the ſpherical body B increaſes equally every where from the center, and therefore that there is an equal preſſure on all ſides at the center, and at equal diſtances from it. Again, in all contrary or oppoſite actions, the weaker muſt give way to the ſtronger, ſo far at leaſt, till they by ſome means become equal. Now, by the action of A the elaſticity of the æther, on the ſide of B next to A, is more leſſened than on the oppoſite ſide; therefore the point of equal preſſure cannot be at the center of the body B, while it is within the ſphere of action of the body A: but a point, between the center of the body B and its ſurface next to A, may be found, in which the contrary actions within the body B in the line A B ſhall be equal: for taking a point, ſo that the quantity of matter between that point and the fartheſt ſurface in the line A B be to the quantity of matter between the ſame point and the ſurface next to A, as the elaſticity of the æther is leſſened more on this ſide than on

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* Paragraph 15.

on the oppoſite, the elaſticity of the æther, and conſequently the oppoſite actions, will become equal.

Now, I think, it clearly appears, that all the phænomena of gravitation neceſſarily follow from the actions of reſiſting, elaſtic, and moving matter, as the cauſes of thoſe phænomena, without ſuppoſing any property or power in matter, which can be called attraction; and that I have thereby freed the mind from that embaraſſment, which the ſuppoſition of an attractive power in matter neceſſarily produces in it. And I hope it will, ſo far at leaſt confirm in the opinion of the candid reader, that theſe phænomena are really produced by the cauſes or actions which I ſuppoſe, and which, I think, I have proved to exiſt. My next attempt ſhall be, to ſhew how the phænomena in the great heavenly bodies neceſſarily and evidently proceed from the ſame cauſes.

CHAP. III.

Of the Motion of the Planets.

1. IT is commonly ſuppoſed, that a certain degree of motion or velocity was communicated to each of the planets at the creation, which has continued ever ſince; and that this motion is in ſome parts of their orbits retarded, by the action of gravitation being contrary to it in their receſs from the ſun; and at other times accelerated, by gravitation, tending the ſame way, or coinciding with it, in their approach towards the ſun. Now, ſuppoſing (but not admitting to be true) that, by theſe contrary actions, a planet from the perihelion to the aphelion run one half of its orbit; yet it can never by the ſame cauſe alone, move a ſecond time in the ſame orbit; but muſt continually from the time it paſſes the aphelion, approach nearer and nearer to the ſun, till at laſt they unite. For, ſince by ſuppoſition, the motion of the planet is only accelerated by gravitation, whatever degree of velocity it acquires from this cauſe alone, the direction of that motion being to the ſun, the planet muſt always at the ſame time be compelled nearer and nearer to the ſun, and the nearer it comes to the ſun, a much

greater force is requiſite to divert it from that direction, no other cauſe being aſſigned to alter the direction or tendency which it acquires by gravitation towards the ſun. The increaſe of velocity alone can never alter the direction, no more than the increaſe of the velocity of a ſtone, falling to the ground, alters the direction which it received, when it was firſt ſuffered to fall from any height, where its motion was the ſloweſt.

2. It ſeems to be a very unphiloſophical method of reaſoning to ſuppoſe, that motion comes immediately from the Divine Being, or that he is the immediate cauſe of all motion, as he muſt be, if no other thing have the power of moving in itſelf; and therefore, that every new motion, at leaſt, is generated by the immediate power of the Creator. This indeed would make any inquiry into the cauſes of the phænomena ariſing by the motion of the planets very ſhort, by ſaying, that their motions are directed by the Spirit which governs them. The laws of action of ſuch an agent can never be diſcovered by any mathematical inquiry, becauſe quantity is its ſole object. Motion is a property or quality, or more properly an action; we cannot then conceive it without ſuppoſing that it exiſts in ſomething, which has in itſelf the power of moving. This power we have before proved * muſt exiſt in ſomething, which we conceive as quantity, and conſequently as matter; and therefore ſome kind of matter muſt exiſt, having the power of motion in itſelf.

3. Let

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* Chap. I. ſect. 3. par. 3.

3. Let us then try to find what kind of thing it is that produces motion, wherever we can find it. I am of opinion, that we need not be very long on our ſearch. The rays of the ſun or light preſents itſelf to our firſt reflections. When we ſee all animal and vegetable motion, and the fluidity of liquors, increaſed or leſſened, by the approach or withdrawing of the ſun, or by the falling of his rays more perpendicularly or obliquely on the ſurface of the earth, or in greater or leſs quantity, we are naturally led to the rays of the ſun or light as the cauſe of this motion. In the moſt violent and rapid motion produced by fire, we likewiſe diſcover light in proportion to the greatneſs of the fire. The ſame is obſerved in the motion produced by the firing of gunpowder. We ſhall more directly be led to light, as the cauſe of motion in the planets, ſo as almoſt to put it out of queſtion, by conſidering that the motion or degree of velocity in the planets is reciprocally as the ſquares of their diſtance from the ſun: which proves, that whatever it be which puts them in motion, it muſt be by an emanation in ſtrait lines from the ſun *. That the velocity of the planets is in a reciprocal ratio of the ſquares of their diſtance from the ſun, is proved thus: aſtronomers agree, that the ſquares of the periodic times of the planets are in the ratio of the cubes of their diſtance from the ſun, that is, calling their diſtance d and their periods p, $d^3 = pp$, then $dd : p :: p : d$, their periodical

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* Chap. II. par. 1.

dical times are as the ſquares of their diſtances: but their velocities are always reciprocal to their periodical times; as if one man takes double the time to walk a mile that another does, the degree of his velocity or motion muſt be one half of the other's; in the ſame manner, and for the ſame reaſon the velocity of the planets muſt be reciprocal to the ſquares of their diſtance from the ſun.

4. From what has been ſaid, and when the prodigious velocity of light, or degree of its motion, ſhall be conſidered; the vaſt number of rays which fall upon every the leaſt point, at the diſtance the earth is from the ſun, ſo that, by the reflection of light from that point in all directions, it can be ſeen at a moderate diſtance, from every point round it; that the light of the ſun evidently extends to all the planets, becauſe they are diſtinctly ſeen by the light of the ſun reflected from them, notwithſtanding that it moves in ſtrait lines from the center of the ſun, and conſequently the rays are continually diverging from each other, as they move from the ſun, and likewiſe continually diverging from the planets: theſe things being conſidered, I ſay, it cannot be doubted that the degree of motion in the rays is ſufficient to give motion to the planets. Again, if light give motion, the degree of motion communicated muſt be in a ratio reciprocal to the ſquares of the diſtances from the ſun, as the motion of the planets is found to be, and ſince every thing in motion communicates its action or motion to every other thing,

to which it is contiguous; from all theſe taken together, it becomes as evident, I think, as any thing of the kind can be, that the planets receive their motion from the rays of light; eſpecially when nothing elſe is diſcovered to which their motion can be attributed. If, beſides this, it evidently appears that all the phænomena of the motion of the planets can be clearly explained, from this ſuppoſition of their receiving their motion from the particles of light, as I hope to do in what follows, all doubt muſt be removed of light's being the cauſe of their motion. And if, upon conſidering the general phænomena or appearances in nature, it appears that motion every where proceeds from light, and nothing elſe can be found to give motion, it will follow, that motion is the eſſential property or action of light, and conſtitutes its nature, by which it is diſtinguiſhed from every thing elſe.

5. By the impulſe or communication of motion from the rays of light alone, it is evident, that the direction of it can only be in ſtrait lines from the ſun; and therefore there muſt be ſome other power or force, which changes that direction, and makes the planets move in circular lines, or in their orbits round the ſun. This, it is now generally agreed, is done by gravitation towards the ſun; that is, as I think, by the reaction of the æther (as deſcribed in the preceding chapter) round the ſun: becauſe no other power can be diſcovered, and this is ſufficient to produce all phænomena, which ariſe from the cauſe, whatever

ever it be, which keeps the planets from flying off in ſtrait lines, as they muſt do if nothing were to hinder them. If the orbits of the planets were arcular, and they are nearly ſuch, the direction of this power muſt be to the ſun, becauſe in this caſe the direction muſt be oppoſite and equal to that which moves them in ſtrait lines from the ſun; and though their orbits be not perfect circles, yet it will be afterwards ſhewn, that the force which keeps the planets in their orbits is directed to the ſun. Then by the preceding chapter, there muſt be a large quantity of reſiſting matter in the ſun, to cauſe this gravitation to it, as well as light, the cauſe of the motion of the planets; and as light does not hinder the action of the reſiſting matter on the æther, but reſiſting matter would ſtop or hinder the action or motion of light, if it were placed in the way of its motion, it is probable, or rather neceſſary, that the reſiſting matter be towards the center, or innermoſt parts of the ſun, and light on the ſurface, or towards the outer parts. So that light may compoſe as it were the ſhell, and reſiſting matter the kernel, of the ſame body. What it is that hinders light from flying off in an inſtant, as in fired gunpowder, muſt be the ſubject of ſome future inquiry. It muſt at preſent be ſuppoſed, or required to be granted, that very thin ſurfaces of light, ſucceſſively and continually, fly off from the body of the ſun, as light does from the flame of any thing on fire.

6. If it be aſked, why motion may not be ſuppoſed to be communicated by the ſun to the æther, and by it continued to any diſtance, as the action of reſiſting matter is? I anſwer, that though the æther may receive the action of motion, and communicate it to ſome diſtance (for from thence ariſes perhaps that kind of action which produces the ſenſe of heat) this communicated action alone would not be ſufficient to produce all the phænomena in motion, as will eaſily appear by conſidering the preceding chapter. Beſides, the different refrangibility of different particles of light, and the ſenſe of colours, which they ſeverally excite, ſhew that the rays of light are tranſmitted from the ſun to the place where they are refracted, and thence to our organs of ſenſe: as Sir *Iſaac Newton* fully proves.

7. There are then two actions in ſtrait lines from and to the ſun; the one, by which every thing is driven from the ſun; the other, by which every thing ſeems to be attracted to the ſun, with a force likewiſe reciprocal to the ſquares of the diſtance from the ſun. The one may be called a *centripetal* force, and the other *centrifugal*. Both the centripetal and centrifugal actions, if only the motion to and from the ſun be conſidered, ariſe from the ſame agent; that is, the centrifugal from the immediate action of light on the body or thing driven from, or which ſeems to fly from the ſun; and the centripetal from the reaction in the æther of the

action of light on the ſame body, in a contrary direction: for from the action of reſiſtance of itſelf no manner of motion can ariſe †, and in this place the continuation of motion by the action of reſiſtance, or that power by which it continues in the ſame ſtate, is not conſidered. Then theſe contrary motions cannot deſtroy each other, becauſe motion cannot be negative to motion; the reſiſting power is only ſo *: the oppoſite actions are become negative to each other in the direction only of the motion, by the reaction of the æther and of the motion in the body impelled. Therefore a body or thing impelled by theſe contrary motions cannot remain at reſt, but muſt move in a direction which will be in a certain ratio to the difference of theſe oppoſite actions: that is, if the centripetal and centrifugal force be equal, the body impelled will move in a line equally inclined to both directions, and therefore muſt move at right angles to the directions of the impelling actions. For example, let *S* (fig. 8.) be the ſun, *P* a planet equally impelled by the centripetal and centrifugal actions, and *S P* the line of their oppoſite directions; then the planet will move in the line *P b* at right angles to the line *S P*: and if theſe contrary equal actions, or impulſes, be renewed every moment, the planet will deſcribe a circle round the ſun. But if the centripetal action be ſtronger than the centifugal, *P* will move in a line as *P c* inclined

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† Chap. II. par. 12.

* Chap. I. ſect. 3. par. 4.

clined to the direction *P S:* and if theſe contrary unequal actions be renewed every moment, the planet will deſcribe a kind of ſpiral, always approaching to the ſun: and if the centrifugal be ſtronger than the centripetal, it will move in a line *P d* declined from *S P*, and by the actions being repeated every moment, deſcribe a kind of ſpiral continually receding from the ſun. Theſe muſt be the caſes, if the two oppoſite actions decreaſe or increaſe by equal differences in the ſame ſpace from the ſun. The direction or angle with the line to the ſun muſt remain always the ſame, the velocity will only alter reciprocally to the ſquares of the diſtance from the ſun.

8. It may be objected, ſince it is here ſuppoſed, that the direction of gravitation is directly towards the ſun's center, and the direction of light or of motion is directly from it, theſe contrary actions muſt deſtroy each other when equal; as when the weight of any body is equal to the force that would move it, it remains at reſt: or if the force of gravitation were leſs, yet it could only move in a direction from the ſun with a degree of motion equal to the difference between the two oppoſite forces. To remove this objection, it may be neceſſary farther to conſider the difference between the action of a body, which has only motion communicated to it, and the action of a thing (or particle of light) to which the

power of moving or action of motion is eſſential. A body which has motion communicated to it, can receive it in one direction only, and therefore can only communicate it in that direction; becauſe it can communicate nothing but what it has received: but all directions are equal to the thing to which the power of moving is eſſential; what determines its motion to any one direction, is only the reſiſtance to its motion being leſs in that direction, than in any other. Thus the inſtantaneous exploſion of light, from all parts of gunpowder, gives motion to a ball, in whatever direction the leaſt reſiſtance is found, for example, in whatever direction the bore of the gun barrel is placed. Now the motion of any planet is oppoſed in its motion from the ſun, by a greater reaction of the motion communicated by the æther on its ſide fartheſt diſtant from the ſun, than on its ſide neareſt the ſun: but the reactions of the æther on the oppoſite ſides of the planet equidiſtant from the ſun, are equal, and being contrary to each other, deſtroy the effect of each other, and conſequently there is no reſiſtance in that direction: the motion is preciſely as much increaſed on the one ſide, as it is leſſened on the oppoſite; therefore the planet will move at right angles to the direction of gravitation, when the contrary actions are equal, with all that degree of motion which it receives from the ſun. The motion of the planet therefore, in this caſe, muſt be in the plane perpendicular to the line connecting the centers of the ſun and planet: why it is in any particular direction in that plane, for example, from weſt to eaſt, will be ſhewn afterwards.

9. But there may be two or more ſeries of progreſſions, increaſing or decreaſing as the ſquares of arithmetical proportionals, or as the ſquares of their diſtance from any certain point; and yet the difference between the terms of each of theſe two ſeries, at every diſtance, be different the one from the other. Let there be two triangles (fig. 9.) A H I and *a h i*, of equal lengths or diſtances A H, *a h*; and let them be equally divided into four parts A B, B D, D F, F H, and *a b*, *b d*, *d f*, *f h*: Suppoſe that two bodies A *a* move the ſame diſtances A H, *a h*, and at every equal diſtance acquire a celerity as B C, *b c*; D E, *d e*, F G, *f g*; H I, *h i*; reſpectively: It is plain, that the celerities in both are increaſed as their diſtances; for B C to D E as A B to A D; and ſo likewiſe as *b c* to *d e* ſo *a b* to *a d*, and ſo on: and yet the celerity acquired at each diſtance, will be double in the one of what it is in the other; and the ſpaces run by theſe celerities, will be every where in both reſpectively, as the ſquares of the diſtances.

10. Since it appears, from what was obſerved in the firſt paragraph of this chapter, that a planet would continually deſcend, or move by the force of gravitation towards the ſun, from the aphelion, till it reaches the ſun, unleſs there were ſome other power to prevent it, or to alter the direction which it has received towards the ſun.

Now if the degree of motion, impreſſed on the planet by the rays of light, be ſuch, that the differences of the increaſe at equal diſtances, impreſſed by the action of light, be greater than the increaſe of the differences of gravitation at the ſame diſtances, the action of light may at ſome diſtances be ſufficient to ſtop the deſcent or motion of the planet towards the ſun. The velocity of any body, moving in a circle round the ſun, or in any ſuch direction, wherein its action or motion is not oppoſed by the reaction of the æther, or by gravitation, is every where as the action of light at that diſtance applied to the force of reſiſtance in the moving body. This motion of a reſiſting body becomes a compound action of both moving and reſiſting, and this compound action is called *momentum* by Sir *Iſaac Newton*, whereby a body, moving with any degree of velocity, requires a greater force to ſtop its motion, than is requiſite to ſtop any other thing, moving with the ſame degree of velocity, but without reſiſtance. Every then the leaſt increment of velocity, in any body thus moving, will be as this increment of velocity multiplied into the quantity of reſiſting force in the moving body. We have hitherto conſidered the increment of velocity only in the ratio of the diſtance from the ſun, or as a linear quantity; now when the quantity of matter, or the ſolidity of the moving body, makes part of the ratio of motion, the ratio of the increment muſt be as the cube of the linear increment.

Pray, what thing can move that is not a Body?

11. Let the equal quantity by the addition of which the arithmetical progreſſion is formed, by the continual addition of which arithmetical progreſſion, the ſquares of the diſtances are formed, and which equal quantity may be called the differential, or fluxional, or incremental element, in the calculation of the force of light on a reſiſting body at the ſeveral diſtances from the ſun: I ſay, let this quantity or element be to the ſimilar quantity or element, in the calculation of gravitation at the ſame diſtances, as the cube of 2, or 8 is to 2; as in the following numbers:

64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441

0, 4, 16, 36, 64, 100, 144, 196, 256, 324, 400, 484, 576, 676

64, 77, 84, 85, 80, 69, 52, 29, 0, 35, 76, 123, 176, 235

13, 7, 1, 5, 11, 17, 23, 29, 35, 41, 47, 53, 59

6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6

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Where the firſt line of numbers repreſents gravitation reciprocal to the ſquares of the diſtance from the ſun, and is made from an arithmetical progreſſion, whoſe common difference is 2, and which is the element or original point in the calculation or compoſition of the force of the reaction of æther: the ſecond line repreſents the effects of the action of light on a reſiſting body, or of motion in that body or its momentum, every where likewiſe reciprocal to the ſquares of the diſtance from the ſun, made from an arithmetical progreſſion whoſe common difference is 8, the cube of the common linear difference where only diſtance is conſidered: the third line ſhews the difference of the effects of theſe two contrary powers at the ſeveral diſtances from the ſun, or the effects of their joint action on a planet at thoſe diſtances; where it is to be obſerved, that the numbers on one ſide of o are all negative to thoſe on the other ſide, and produce a contrary effect: the fourth line are the differences of the terms of this joint action at the ſeveral diſtances: and the fifth line ſhews the element or differential from which the calculation of this joint action is formed.

12. Now, ſince the centripetal action does not oppoſe the progreſſive circular motion of a planet in its orbit, the ſecond line will repreſent the ratio of velocity of a planet, or of ſeveral planets, moving in a circle round the ſun, at ſeveral diſtances from the ſun. Since the third line of numbers is the differences of the two contrary actions, the centripetal and centrifugal, it ſhews the ratio of the one to the other, at the ſeveral diſtances from the ſun, where o is the ratio of equality, between which and the ſun light is the ſtronger; but on the oppoſite ſide of the diſtance of their equality, or at a greater diſtance, light is the weaker. Theſe contrary powers may be ſo ordered, as in the preceding example they are, that at one certain diſtance their forces may become equal, as in this example at the number 256, and then the planet muſt move at that diſtance in a circle round the ſun, with a progreſſive motion, in the ratio of 256 to any other number of the progreſſions,

where the oppoſite progreſſions may be ſo placed, as to be equal at ſome other diſtance from the ſun: for the place of equality depends upon the manner of placing one progreſſion to the other. The rule for placing them will be ſhewn afterwards. The planet has a centripetal motion at the greater diſtance from the ſun than this place of equality, and a centrifugal motion between this place and the ſun. It appears from theſe numbers, that the effects or force of the centrifugal motion of a planet is every where, to this place of equality, in a ratio reciprocal to the ſquares of the diſtance from the ſun; and the ratio of the effects or force of the centripetal motion of a planet, is reciprocal to the ſquares of the diſtance from the place of equality. This place then may be properly called *the center of the centrifugal and centripetal motions of a planet.* The fourth line of numbers ſhews the ratios of velocity by which a planet recedes from the ſun, by the power of reſiſting in itſelf: for, ſince a planet moves in its circular motion, with all the velocity that can be communicated to it, at its ſeveral diſtances from the ſun, no more motion remains to be continued or can be continued by its reſiſting power, dictinctly from the moving power, but only the difference of the velocities communicated at the ſeveral diſtances, and that after the velocity communicated is leſſened by the reaction of the æther, at thoſe ſeveral diſtances. It likewiſe can only be in a direction to or from the ſun's center, becauſe in the circular motion nothing remains to be continued. And the laſt or fifth line ſhews the differences of theſe increments, which is every where the ſame, and is equal to the difference of the elements of the abſolute centrifugal and centripetal actions, viz. 6=8—2.

13. So far the two contrary or oppoſite centrifugal and centripetal actions upon any thing have been conſidered, without conſidering the power or action which a planet has in itſelf; but as the planets conſiſt of ſome quantity of reſiſting matter, which has a power of continuing any action it receives*, by its oppoſing all change of its preſent ſtate, the effects of this power muſt likewiſe be more particularly conſidered, before we can have a perfect or compleat conception of the motion of a planet. It will be more eaſy to form an idea, how far this power affects the motion of a planet, by conſidering its effects in projectiles: and for this reaſon, it is proper to premiſe ſomething on the nature of projectile motion.

14. A ball of any quantity of reſiſting matter being projected or thrown, in a line perpendicular to the horizon, or in a line paſſing through the center of the earth, by the hand, or by the force of gunpowder, or by any other power which communicates motion to it, will, in the firſt inſtant, acquire a degree of motion or velocity, equal to the difference of force or degree of action between the degree of motion communicated, and the reſiſtance or quantity of matter in the ball, to which the

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[*] Chap. I. ſect. 2.

motion is communicated. Thus if a degree of motion communicate a velocity as 20 to a ball of one pound weight, the ſame degree of motion will only communicate a velocity as 10 to a ball of two pound weight. This communicated motion, if all other actions ſhould ceaſe, would continue the ſame without any diminution, by the power in the ball, reſiſting or oppoſing all change of its preſent ſtate, and the ball would fly upwards with the ſame velocity eternally, or as long as all other action ceaſed. If the æther were on the ſide of the ball fartheſt diſtant from the earth, and no æther on the ſide next the earth, the ball would communicate the whole degree of its motion to the æther, and the æther by its reaction (always equal to the action which it receives) would entirely deſtroy the motion of the ball, and its motion would that moment ceaſe*. If the ball be every where ſurrounded with æther, it will communicate its action to the æther all round it; and if the elaſticity of the æther be equal all round it, the ball will have the ſame degree of action communicated to it by the reaction of the æther on the ſide next the earth, that is taken from it by the reaction of the æther on the oppoſite ſide; conſequently, the ball will continue to move with the ſame degree of velocity which it firſt received. But if the elaſticity of the æther be greater on the ſide of the ball fartheſt diſtant from the earth, than it is on the ſide of the ball neareſt the earth (as it really is) then the velocity of the ball will be continually leſſened by the differences between the elaſticity of the æther on the one ſide and the other of the ball. But the differences of the differences between the elaſticity of the æther in equal diſtances, at ſeveral diſtances are arithmetical proportionals, decreaſing as the diſtance from the earth increaſes; and therefore the velocity of the ball will continually decreaſe in an arithmetical proportion, and at laſt its velocity will become equal to 0. For example, if a ball when firſt ſhot from a cannon in a perpendicular direction move with a velocity as x, and aſcending a certain diſtance as a rod, or in a certain time as one minute, the velocity becomes $x-7$, then the next rod or equal diſtance or equal time it will become $x-7-5$, and ſo continually decreaſing in an arithmetical progreſſion whoſe common difference is 2, till the ball ceaſe to aſcend, which will happen when the velocity or degree of motion in the ball becomes leſs than the difference between the elaſticity of the æther on the one ſide of the ball and its oppoſite ſide. For example, the degree of velocity in a ball aſcending, ſuppoſing the time or diſtance of its aſcent divided equally into 4, and the velocity $x = 16$ at the ſeveral diſtances or times, will be as the following ſeries.

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* Chap. I. ſect. 3. par. 6.

1 $x-7=9$
2 $x-7-5=4$
3 $x-7-5-3=1$
4 $x-7-5-3-1=0$

If no motion were communicated to the æther from any other moving thing, the ball would remain at reſt where it loſes its projectile motion; but as the æther has, in fact, motion perpetually communicated to it from light, the ball by its gravitation will then acquire a new motion with a contrary direction, that is, downwards: and as the difference of elaſticity in the æther continually increaſes, the velocity of the ball muſt continually increaſe, in the inverſe ratio in which it decreaſed in aſcending, ſo that when it comes to the ground, it will have the ſame velocity it had when ſhot from the cannon; for the velocity increaſes and decreaſes by the ſame common quantity. So that the velocity with which a ball thus runs, from the point where it begins to deſcend, will be at ſeveral diſtances as the ſquares of thoſe diſtances from that point; and the diſtances to which ſeveral balls will aſcend, are as the ſquare roots of the velocity with which they ſeverally begin their aſcent. This is the general law of projectiles (abſtracted from the reſiſtance of the atmoſphere) when there is no new degree of motion communicated to the projectile, but at the firſt inſtant.

15. If a planet were to be projected from the ſun, the caſe would be otherwiſe, becauſe it would have a new motion (beſides that of the reaction of the æther) communicated to it every inſtant, by the rays of light; and this new degree of motion, which is added, is always re-

ciprocal to the ſquares of the diſtance of the planet from the ſun: ſo that the degree of centrifugal motion in the planet, will every where be different from that of a projectile as above deſcribed; but notwithſtanding this, the planet's velocity will every where be reciprocal to the diſtance. I call the degree of motion communicated by the rays of light, abſtracted from any conſideration of the contrary action of the æther, the *abſolute centrifugal force*; and in the like manner, the action of the æther, without conſidering the contrary action of light, I call the *abſolute centripetal force*.

16. The third line of figures, in the eleventh paragraph, being the difference of the effects between the abſolute centrifugal and centripetal forces, ſhews that a planet, if it were not endowed with the power of reſiſting, would ſtop when it came to the place where the abſolute centrifugal and centripetal actions are equal, or that place which I call the center of a planet's centrifugal and centripetal motion. But as a planet is endowed with the reſiſting power, when it comes to the center of the centrifugal and centripetal motion of the planet, or where the centripetal and centrifugal actions are equal, it will ſtill move from the ſun, by the force of itsreſiſting power, as in the caſe of a projectile. And the degree of motion or velocity every where, which the planet will retain by its reſiſting power, will be every where as the difference of the difference of the abſolute centripetal and centrifugal actions, or as the fourth line of numbers

in paragraph eleventh; which appear to be arithmetical proportionals, decreaſing as the diſtance from the ſun †; and the celerity or velocity with which it will go from the center of its centrifugal and centripetal motions; either way, will decreaſe reciprocally as the ſquares of the diſtances from that center. This will be eaſily underſtood by conſidering the caſe of projectiles in the fourteenth paragraph. And after the centrifugal motion has intirely ceaſed, it will return with the ſame velocity it moved from the ſun, at the ſeveral diſtances, till it come to the place where the abſolute centrifugal and centripetal actions were equal, at which place it will have the ſame degree of velocity it had when it aſcended at that place; and this for the ſame reaſon that a projectile returns to the ground with the ſame velocity that it was thrown from it; which will eaſily appear by following the ſame method that was uſed in the caſe of a projectile. In the caſe of a projectile, the reſiſtance which the projectile meets, when it comes to the ground, entirely deſtroys its motion: but a planet, when it comes to the place where the abſolute centripetal and centrifugal powers were equal, meets with no ſuch reſiſtance; however, it meets with a greater than it did before, for from this place towards the ſun the centrifugal force is greater than the centripetal. And as by the centripetal force being greater than the centrifugal beyond this place was the reaſon that the planet's centrifugal velocity conti-

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† Par. 12.

nually decreaſed, ſo in like manner the centripetal motion, after it paſſes the place where the abſolute centripetal and centrifugal powers were equal, muſt continually decreaſe till it become = 0, becauſe the centrifugal is greater than the centripetal: and ſince the difference between the terms of the planet's centrifugal and centripetal motions every where increaſe and decreaſe equally, its velocity will likewiſe continually decreaſe or increaſe equally: conſequently, a planet will deſcend exactly as far below the point, where the centrifugal and centripetal motions were equal, as it aſcended above it; and ſo it will continue to deſcend and aſcend, with an oſcillatory motion like a pendulum, ſo long as the abſolute centripetal and centripetal actions continue, that is, as long as the light of the ſun and the æther ſhall continue, together with the planet's power of reſiſting. The diſtance to which this centrifugal and centripetal motion in the planet extends from its center, or place where the abſolute centrifugal and centripetal actions are equal, may be called the *limits of the planet's centrifugal and centripetal motion.*

17. As this oſcillatory motion, by which a planet in its circuit round the ſun alternately approaches towards the ſun and recedes from it, makes a principal part in the phænomena of the motion of the planets, the true cauſe of which (ſo far as I know) has not been hitherto explained, it is neceſſary to give as clear a conception in what manner this is done as poſſible; and for this purpoſe

I ſhall

I ſhall farther explain it by an illuſtration in numbers. It depends intirely upon the following principles: viz. the motion continued by the reſiſting power of the planet (or what may be called its projectile motion) is increaſed by light from the ſun in an arithmetical progreſſion decreaſing from the ſun; ſo that it goes farther from the ſun by its reſiſting power than it could by the action of light alone. This motion from the ſun is oppoſed by the reaction of the æther likewiſe in an arithmetical progreſſion decreaſing from the ſun. But the progreſſion which adds to its motion from the ſun, decreaſes or increaſes faſter than that which oppoſes it, the one by 8 the other by 2, and the difference of the ratios of their increaſe or decreaſe is 6. Again, when the planet approaches the ſun, from the center of its oſcillatory motion (or from the center of its centripetal and centrifugal motion) its motion is increaſed by the reaction of the æther, and oppoſed or leſſened by the action of light. Now ſuppoſe the velocity which a planet by its reſiſting power continues, or the velocity communicated to the planet at that diſtance, to be 32; the action of the æther and of light are, by ſuppoſition, both equal to it at that diſtance: then the increaſe and decreaſe of the planet's motion, ſuppoſing the increments or differentials of the increaſe to be in an arithmetical progreſſion, will be in the ratio of the following numbers.

The

32	32	32	32	32	32	The velocity continued by the planet's reſiſting power.
32	24	16	8	0	—8	Force or action of light at the ſeveral diſtances.
64	56	48	40	32	24	The joint action of theſe two powers.
32	30	28	26	24	22	The action of the æther at the ſeveral diſtances
32	26	20	14	8	2	The effects of theſe oppoſite actions on the planet in going round the ſun.
32	32	32	32	32	32	The reſiſting action.
0	2	4	6	8	10	The action of the æther at the ſeveral diſtances.
32	34	36	38	40	42	The joint action of the æther and reſiſting power.
0	8	16	24	32	40	The oppoſite action of light.
32	26	20	14	8	2	The effects of theſe two actions in its approaching towards the ſun from the center of its oſcillatory motion.

It is well known, that any ſeries of decreaſing numbers, after the ſeries is brought to nought 0, may be continued to any length, by adding negative numbers increaſing from 0 in the ſame ratio the others decreaſed.

It cannot be difficult to conceive, how the planet, when it comes to that diſtance from the center of its centrifugal and centripetal motions, at which the action of the æther (or the planet's gravitation) is equal to the joint actions of light, and the planet's reſiſtance, can go no farther from the ſun, as in the example 22 + 8 + 2 = 30 + 2 (the quantity of reſiſtance left) = 32. Again, when the action of light becomes equal to the joint actions of the planet's reſiſtance and of the æther, (or of the planet's gravitation) it can go nearer the ſun, as 10 + 32 = 40 + 2. As ſoon as the action of the planet's reſiſtance ceaſes (as it always muſt when any contrary action is equal to it) the action of the æther, ſuperior to the action of light in one caſe, and the action of light ſuperior to that of the æther in the other caſe, muſt make the planet return as in the following numerical example of the planet's return from its greateſt diſtance from the ſun.

32	32	32	32	32	32	The planet's reſiſtance.
22	24	26	28	30	32	The action of the æther at the ſeveral diſtances.
54	56	58	60	62	64	Their joint action.
—8	0	8	16	24	32	The action of light at the ſame diſtances.
62	56	50	44	38	32	The ſuperiority of the action of æther.
64	64	64	64	64	64	The greateſt joint action of reſiſtance and the æther.
2	8	14	20	26	32	The ratio of the velocities in which a planet will deſcend to the center of its centrifugal and centripetal motion.

I have put the examples here in ſeries of arithmetical proportionals, as leſs complicated, and thereby more eaſily conceived; but the reaſoning is the ſame, when the oppoſite actions increaſe or decreaſe as ſquares of arithmetical proportionals, as in truth the actions of the æther and of light do. I ſuppoſe it is remember'd that no body or thing, acting only by a force or action communicated to it, ever recovers of itſelf any part of that action which it loſes by a contrary or oppoſite action.

18. If the place where the abſolute centripetal and centrifugal actions become equal, or the center of the planet's centrifugal and centripetal motion be nearer the ſun, as in the firſt of the following numerical examples, or be farther from the ſun, as in the ſecond, the degree of velocity,

city, which the planet's reſiſting matter continues, will be the ſame at the ſame diſtances from the ſun in both caſes, and generally in all caſes; that is, always reciprocal to its diſtance from the ſun *.

First numerical example.

81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441
0, 4, 16, 36, 64, 100, 144, 196, 256, 324, 400, 484, 576

81, 96, 105, 108, 105, 96, 81, 60, 33, 0, 39, 84, 135
—15,—9,— 3, 3, 9, 15, 21, 27, 33, 39, 45, 51
6, 6, 6, 6, 6, 6, 6, 6, 6, 6, 6
36

Second example.

36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289
0, 0, 4, 16, 36, 64, 100, 144, 196, 256, 324, 400

36, 49, 60, 65, 64, 57, 44, 25, 0, 31, 68, 111
13,--11,—5, 1, 7, 13, 19, 25, 31, 37, 43
2, 6, 6, 6, 6, 6, 6, 6, 6, 6
28

The ſmall variations from the general rule which appear in the numbers, are owing to the place where the limits of the planet's centrifugal and centripetal motion falls. Sometimes they fall exactly at the end of the diſtance aſſumed in the calculations, at other times in ſome middle part of thoſe diſtances.

19. The velocity with which a planet paſſes from the center of its centrifugal and centripetal motion to either of its limits, is reciprocal to the ſquares of the diſtance from

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* Par. 12.

from the ſun: for it is made up of the ſeveral velocities which at equal diſtances are ſquares of arithmetical proportionals. Thus in the triangle A H I (fig. 9.) compoſed of the lines B C, D E, F G, H I, (or more properly of an infinite ſeries of arithmetical proportionals) the content or area of the triangle is equal to the ſum of theſe, equal to A H multiplied by the half of H I. If in the place of arithmetical proportionals decreaſing to o their ſquares were taken, the area would be equal to A H multiplied by $\frac{1}{3}$ H I. But the halfs or thirds are in the ſame ratio with their wholes, as A B to A D as B C to D E; therefore the content of the triangle is in the ratio of H I ſquared, or in the ratio of A H ſquared, both being ſimilar ratios. The velocity then with which the length of the ſide A H is run, or the velocity with which a planet runs the diſtance from the center to either limit of its centrifugal and centripetal motion, is always in a ratio of the ſquare of the diſtance from the center to the limit, that is, in a ratio reciprocal to the ſquare of the diſtance of this center from the ſun: and the time in which it runs this diſtance, being reciprocal to the velocity, muſt be in the direct ratio of the ſquare of the diſtance of this center from the ſun. For, if the velocities increaſed only as the diſtances run, the times in which they are run would be all equal: but ſince the velocities increaſe as the ſquares of the diſtances, the times in which they are run will be reciprocal to the ſquares of the diſtances; for a greater velocity makes the time leſs. Since the diſtance of the center of a planet's centrifugal and centripetal motion is the ſame with the mean diſtance at which it performs its circuit round the ſun *, the time, in which a planet runs from the center of its centrifugal and centripetal motion to either limit, has likewiſe the ſame ratio to the periodical time in which it performs its circuit round the ſun. For the velocities of a planet, in both its circular motion and centrifugal or centripetal motion, are ſeverally at the ſeveral diſtances from the ſun, reciprocal to the ſquares of the diſtance from the ſun, and their reſpective periodic times directly as the ſquares of the diſtances from the ſun.

20. It appears from the numerical examples in paragraphs 11 and 18, that the ratio of the ſquare of the diſtance from the center of a planet's centrifugal and centripetal motion to either of its limits, and conſequently the ratio of its velocity, with which it moves from that center to either of its limits, is preciſely quadruple the ratio of the velocity with which the ſame planet performs its circuit round the ſun: and, as the times are reciprocal to the velocities, a planet will move four times preciſely from the center of that motion to one of its limits in the time that it performs its circuit round the ſun; and this muſt always be the caſe when the equidiſtant terms from 0 of the two ſeries are the one quadruple of the other. In the firſt example given, the ratio of the diſtance between

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* Par. 3.

tween the center of the planet's centrifugal and centripetal motion and its limit is 32, and the ratio of the velocity of the planet with which it performs its circuit at that diſtance from the ſun is 256, the ſquare of 32 is 1024, which divided by 4 is 256. In the ſecond example, the ratio of the diſtance between the center and limit is 36, and the velocity of the planet 324, the ſquare of 36 is 1296, which divided by 4 is 324. And in the third example, the ratio of the diſtance is 28, the ratio of velocity 196, the ſquare of 28 is 784, which divided by 4 is 196.

21. Thus, if the line A B (fig. 10.) be the circle of a planet's mean or equal motion, it will likewiſe be the place of the center of its centripetal and centrifugal motion, which center muſt be ſuppoſed to move along equally, as the planet is ſuppoſed to do, by its progreſſive motion. Let this line be divided into four equal parts at C D E, and theſe again ſubdivided into any number of equal parts, as 2, 3, 4, *&c.* and at every one of theſe ſubdiviſions, let lines be erected perpendicular to the line A B, whoſe lengths increaſe in a decreaſing arithmetical progreſſion, or as lines whoſe differences are as the numbers, for example 7, 5, 3, 1, to the line C F, erected on the extremity of the firſt quarter of the circle or line A B, from thence let the lines continually decreaſe in their lengths, ſo that their differences be as the arithmetical progreſſion 1, 3, 5, 7, and that at D the ſecond quarter the length be = 0;

$= o$, theſe decreaſing and increaſing arithmetical progreſſions, will be as the centripetal or centrifugal force of the planet, at the ſeveral diſtances from the center of its centripetal and centrifugal motion in the line A B. Again, let the perpendicular lines increaſe on the lower ſide of the line A B, as they did in the firſt quarter, from D to E, and decreaſe again from E to B as they did the in ſecond quarter. For example, let the perpendicular lines in the firſt quarter being divided into four equal parts, be 7,12,15,16, and in the ſecond 15, 12, 7, 0; in the third as in the firſt, and in the laſt quarter as in the ſecond; then the curve line AFDGB will be the line in which the planet in this caſe would move; and ſuppoſing A B to be the circumference of a circle, the point B muſt coincide with A. However, this curve line does not truly repreſent a planet's orbit, it is only ſuppoſed to aſſiſt the imagination in forming a true idea of a planet's orbit and motion, which will be afterwards ſhewn.

22. In the preceding paragraph it was ſuppoſed, that the progreſſive motion of a planet in its orbit is every where equal, whereas it is otherwiſe in reality. For, as the planet recedes from the ſun, its motion in its orbit is retarded and when the planet accedes, it is accelerated. The planet's centrifugal and centripetal motion decreaſes or increaſes every where reciprocally to the ſquares of its diſtance from the center of this motion, and the progreſſive motion is every where reciprocal to the ſquares of the

planet's diſtance from the ſun. While a planet, by its centripetal or rather centifugal motion, moves from the limit of that motion which is neareſt the ſun, this motion increaſes directly as the ſquares of the diſtance from the limit, and the planet's progreſſive motion decreaſes reciprocally as the ſquares of the ſame diſtances, till it reach the center of its centrifugal and centripetal motion. From thence, the planet's centripetal motion increaſes reciprocally as the ſquares of the diſtances, and the progreſſive motion decreaſes in the ſame ratio, as likewiſe does the planet's centrifugal motion. When, by its centripetal motion, a planet moves from the limit of this motion fartheſt diſtant from the ſun to the center of its centrifugal and centripetal motion, its centripetal motion decreaſes in a reciprocal ratio to the diſtances from this center, and the progreſſive motion is accelerated in the ſame ratio; but when it paſſes this center, its centripetal motion decreaſes reciprocally as the ſquares of the diſtances from this center, and the progreſſive motion increaſes directly as the ſquares of the diſtances from that center: ſo that the centrifugal and centripetal motions are always increaſing or decreaſing while the progreſſive motion is decreaſing or increaſing. The planet's motion in its orbit is preciſely as much retarded, when it is beyond the center of its centifugal and centripetal motion, as it is accelerated when it is between that center and the ſun. So that it performs its periodic courſe preciſely in the ſame time it would have done, had it moved in a circle, at the diſtance of the center of its centrifugal and centripetal motion, or at the diſtance at which this center is ſuppoſed to move.

23. Theſe things being premiſed, the motion of a planet in its orbit may be demonſtrated by the following example. Let F (fig. 11.) be the place of the ſun, L the limits of the planet's centrifugal and centripetal motion neareſt the ſun; let L M be the diſtance between the limit and center of the planet's centrifugal and centripetal motion, which in the ſcheme is made the ſine of 30 degrees; with the radius F L draw the ſemicircle L C, the planet in no place can come nearer the ſun than this circle; with the radius F M draw the ſemicircle M A *f*, the center of the planet's centrifugal and centripetal motion will move in the circumference of this circle; divide the diſtance M L between the center and limit of the planet's centripetal and centrifugal motion into any number of parts, which ſhall be thought proper, ſuppoſe into four parts, as at *a*, *b*, *c*, in ſuch manner, that the diſtances M *c*, M *b*, M *a*, M L be reciprocal to the ſquares of the diſtances from M, which ſince M L is 30 degrees will be $La = 1^\circ\ 52'\ 30''$, $ab = 5^\circ\ 37'\ 30''$, $bc = 9^\circ\ 22'\ 30''$, and $cM = 13^\circ\ 7'\ 30''$*, that is, make $Mc = S\ 13^\circ\ 7'\ 30''$, $Mb = S\ 22^\circ\ 30'\ 0''$, $Ma = S\ 28^\circ\ 7'\ 30''$, $ML = S\ 30^\circ\ 0'\ 0''$. At the diſtance F *a*, F *b*, F *c*, and from the center F, draw the arches

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* Chap. II. par. 19.

a, *b*, *c*, repreſented by prickt lines: divide the quadrant of the circumference, in which the center of the planet's centrifugal and centripetal motion moves, into as many parts as the diſtance between the limit and that center was divided, into four parts in the preſent caſe, each equal therefore to 22° 30′; add to each of them ſeverally the above diviſions of the 30 degrees, the diſtance between the center and limit of the planet's centrifugal and centripetal motion; then the diviſions of this circumference will be 24° 22′ 30″, 28° 7′ 30″, 31° 52′ 30″, 35° 37′ 30″. And, ſince the motion of a planet is always ſlower as it recedes from the ſun, in a ratio reciprocal to the ſquares of the diſtances, place the greateſt diviſion next to M, and ſo on the next greateſt, till the ſmalleſt be placed laſt: then an arch of that circle, or the circle of the mean motion, will be divided in this manner, at *d*, *e*, *g*, and A, and M A will be an arch of 120 degrees $= 90 + 30$. If lines be drawn from F to *d*, *e*, *g*, A, the points *h*, *i*, *k*, A, where theſe lines cut the arches *a*, *b*, *c*, M, conſequently, will be points in the planet's orbit: becauſe the diſtances or arches M *d*, *d e*, *e g*, and *g* A, are in the ratio of the planet's progreſſive motion, and the diſtances L *a*, *a b*, *b c*, and *c* M, in the ratio of its centrifugal motion. After this manner, as many points of the planet's orbit may be found as ſhall be deſired, for the firſt quadrant of a planet's orbit.

24. Set from the ſun's place F towards *f*, in the line of the apſides of the planet's orbit, the diſtance between the limits of the planet's centripetal and centrifugal motion, equal to twice the diſtance L M, or to twice the S 30 degrees (this diſtance in the preſent caſe coincides with *f*) from *f* as a center, at the diſtance of the center of the planet's centrifugal and centripetal motion from the ſun F M, deſcribe the arch A B, from A to the line of the apſides; this arch is equal to the arch M A: divide it from A to B at *x*, *y*, *z*, in the ſame manner A M is divided, from A to M. From *f* likewiſe deſcribe the ſemicircle C D *l*, at the diſtance of the neareſt limit of the planet's centripetal and centrifugal motion from the ſun, or at the diſtance F L, and divide with concentric circles at *f*, the diſtance between this circle and the arch AB, from *l* to B, at *s*, *t*, *v*, in the ſame manner the diſtance L M is divided, from L to M: then from *f* draw the lines *f x*, *f y*, *f z*, the points where the lines *fx* cut the arch *v*, *f y* cuts the arch *t*, and *f z* cuts the arch *s*, will be points in the planet's orbit. For the planet's progreſſive velocity decreaſes from the neareſt limit to the mean diſtance, reciprocally to the ſquares of the diſtances from the ſun, and from the fartheſt limit to the mean diſtance, it increaſes reciprocally as the ſquares of the diſtances from the ſun. Therefore this increaſe and decreaſe will, in both caſes, have the ſame ratio to the increaſe of the planet's centrifugal and centripetal motion from theſe limits to the

mean diſtance. Therefore the arches and ſines repreſenting the ratios of theſe two different increments, muſt at the reſpective diſtances from the mean diſtance have the ſame ratio the one to the other, after the planet has paſſed its mean diſtance, which it had before it came to it. Thus *Md* to *La*, as *Bz* to *ls*, and *de* to *ab*, as *zy* to *st*, &c. This perhaps may become the more evident, by putting *m* for the velocity, with which a planet would in any number of equal diviſions of its orbit, ſuppoſing it moved always at the ſame diſtance, and which is the ſame with which it moves at its mean diſtance. Divide the diſtance between the center and limits of the planet's centripetal and centrifugal motion both ways, into a number of arithmetical proportionals, correſponding to the number of diviſions in its orbit, and increaſing on each ſide from the limit to the center, and converted into the diviſions of a circle as before mentioned, that is, ſines in the one anſwering to degrees of the other. Suppoſe the diviſions of the centripetal and centrifugal motion to be 1, 3, 5, 7; then the planet's velocity in its orbit, and the velocity of its centripetal and centrifugal motion, at the ſeveral diſtances in its moving from the one limit to the other, will ſtand with reſpect to each diſtance, as repreſented in the order of the two following ſeries.

$m+7$,	$m+5$,	$m+3$,	$m+1$,	m,	$m-1$,	$m-3$,	$m-5$,	$m-7$	*The Orbit.*
0	1	3	5	7	5	3	1	0	{ Centripetal or Centrifugal.

where it may be obſerved, that the ſeries repreſenting the planet's motion in its orbit, from the one limit to the other, is part of an arithmetical progreſſion, as it muſt be, where the planet by its centripetal or centrifugal motion is always either approaching or receding from the ſun : and that the ratios of the increments of the velocity in the orbit to the reſpective velocities of the centripetal velocity, are always the ſame, whether the increments of the planet's velocity in its orbit be repreſented by poſitive or negative numbers ; then, ſince the diſtances run in equal times are always proportionable to the velocities, the planet's place in its orbit may be every where found, its mean motion and excentricity being given. Thus, I think, the figure of a planet's orbit and its place in it, at any time, may be eaſily and geometrically diſcovered, from the principles of action which give it motion, and without any aid of the conic ſections; which, in effect, is a ſolution of *Kepler*'s famous problem, which has ingaged the attention of all noted aſtronomers ſince his time.

25. If all the circles in this ſemicircle be compleated, it is evident that the planet's place in the other half of its orbit, may be found in the ſame manner that was uſed for finding it in this firſt half, viz. in the third quadrant of its orbit as in the ſecond, and in the laſt quadrant as in the firſt. Compleat the parallelograms F *b f* G, F *i f* H, F *k f* K, and F A *f* N ; it is evident that the lines F *b*, F *i*, F *k*, and F A, increaſe in the ſame ratio that the

planet's centrifugal motion does, and in a reciprocal ratio to the progreſſive motion of a planet in its orbit; and the lines fh, fi, fk, and $f\mathrm{A}$ decreaſe in the ratio of the progreſſive motion of the planet in its orbit from L towards A, and in a reciprocal ratio to the planet's centrifugal motion in the ſame time. And the diagonals of theſe parallelograms all paſs through the ſame point C. For $f\mathrm{L} = \mathrm{S.}\ 90 - \mathrm{S.}\ 30 + 2\ \mathrm{S.}\ 30 = \mathrm{S.}\ 90 + \mathrm{S.}\ 30$, and $f\mathrm{A} = \mathrm{S.}\ 90 = \mathrm{F\,A}$, but $\mathrm{F\,L} = \mathrm{S.}\ 90 - \mathrm{S.}\ 30$; therefore the one has increaſed by the ſame quantity that the other has decreaſed: and they likewiſe increaſe and decreaſe by an addition and ſubſtraction of the ſame quantity. For if the lines F L and $f\mathrm{L}$ [fig. 12.] the one F L increaſe by a continual addition of arithmetical increaſing proportionals, and the other $f\mathrm{L}$ decreaſe by the taking away the ſame quantities, the one will increaſe as the ſquares of arithmetical proportionals, and the other decreaſe reciprocally to the ſquares of the ſame proportionals: it is eaſy to apply the ſame kind of reaſoning to the ſecond quadrant of the planet's orbit, as appears from the ſcheme. It will likewiſe be eaſy to prove, that the points G, H, K, N, *&c.* are points in the other half of the planet's orbit. If then the angles L F h, h F i, *&c.* be made infinitely ſmall, the planet will deſcribe the curve L, h, i, k, A.

26. It appears then plainly that the orbit of a planet is an ellipſis, tho' I have no where made uſe of the properties of that curve, in demonſtrating the figure which a pla-

net deſcribes in its motion. It is of more uſe to demonſtrate the nature of its orbit immediately from the cauſes of its motion, as thereby the mind receives a clearer conception of the manner in which the power which gives and regulates its motion acts. And I hope I ſhall afterwards ſhew ſome advantages in this method, by its leading to a clearer and more eaſy way of finding a planet's place at any time by calculation than hitherto (ſo far as I know) has been done.

27. It remains next to ſhew, in what manner the planet's place in its orbit is to be reduced to its apparent place in the ecliptic. But before this be done, it muſt be previouſly obſerved, that if a planet were continually to move in the circle, in which the center of its centrifugal and centripetal motion moves, it would always move equally, at no time faſter or ſlower, and would at all times deſcribe arches of that circle, proportionable to the time in which it moves. As a planet moves faſter when it approaches the ſun, and ſlower when it recedes from the ſun, the equal motion which it would have had by moving conſtantly in this circle, being the mean between its quickeſt and ſloweſt motion, is called the *mean motion* of a planet. And the ratio, in which a planet moves ſwifter or ſlower, as it recedes from the circle of its mean motion, is called *the equation* of the mean motion, which being added to or ſubſtracted from the mean motion, as the planet is on the one ſide or the other of the circle of mean motion,

gives the planet's true motion at any time. For example, the circle M A *f* is the circle of mean motion, then if a quadrant of this circle be divided into four equal parts, each containing 22° 30′, the planet moving in this circle would deſcribe each of theſe diviſions in the ſame time: but as it is neareſt the ſun at L, and comes nearer and nearer to the circle of mean motion, as it moves from L towards A, it will in that ſpace move faſter than it would have done, had it moved in the circle of mean motion, ſo as to gain (on the ſuppoſition we have gone upon) 30 degrees more than a quadrant. And, as the celerity of the planet increaſes as its diſtance from A, the equations of the mean motions muſt accordingly continually increaſe, in the ſame ratio that the planet increaſes. Thus in the firſt diviſion next to A, the equation for the planet's velocity is 1° 52′ 30″, and therefore the planet, in this laſt diviſion, will gain ſo much, or it will run 24° 22′ 30″. The equations for the half of the quadrant of mean motion next to A are $1^\circ\ 52'\ 30'' + 5^\circ\ 37'\ 30'' = 7^\circ\ 30'$, then the planet will run in this half 52° 30′. In three quarters of the quadrant of mean motion the equations are $1^\circ\ 52'\ 30'' + 5^\circ\ 37'\ 30'' + 9^\circ\ 22'\ 30'' = 16^\circ\ 52'\ 30''$, and the planet will run 84° 22′ 30″. And in running the whole quadrant of mean motion it will gain $1^\circ\ 52'\ 30'' + 5^\circ\ 37'\ 30'' + 9^\circ\ 22'\ 30'' + 13^\circ\ 7'\ 30'' = 30^\circ$ the greateſt equation, and therefore in running from L to A will gain ſo much or run 120 degrees.

28. As aſtronomers find it moſt convenient to refer all their calculations to parts or diviſions of a circle, and ſince it is evident from what precedes, that the motion of a planet has every where a reference to C, the middle point between the apſides of a planet's orbit, a circle in the plane of the planet's orbit is choſen for that purpoſe, of which the line connecting the apſides is the diameter. This circle in the plane of the earth's orbit, is called the ecliptic. Now to transfer the planet's place to the circle L E *l*, of which C is the center, divide the quadrant L E into four equal parts at 2, 3, 4, as the quadrant of the planet's orbit was divided in the preceding example, and draw from the center C, through the diviſions, to the circle of the mean motion, the lines C 2 *m*, C 3 *n*, C 4 *o*. Then in the triangle F C *m*, the ſides F *m* (equal to the radius or diſtance of mean motion) F C (equal to the ſine of 30 degrees) and the angle F C *m* (by conſtruction equal to 22°30′) are all given, the angle F *m* C may be found thus: as radius is to the ſine of 22° 30′, ſo is the ſine of 30° to the angle F *m* C. But the angle M F *m* (the meaſure in the circle of mean motion of 22° 30′ in the ecliptic) is equal to the angles F C *m* + C *m* F: for example.

	°	′	″		
S.	30	00	00	9.698 9700	
S.	22	30	00	9.582 8397	
				19.281 8097	
Radius. - - -				10.000 0000	
	11	01	52		
	22	30	00	9.281 8097	equal to the angle C *m* F.
	33	31	52		equal to the angle M F *m*.
	35	37	30		equal to the angle M F *d*.
	2	05	38		equal to the angle *m* F *d*.

In like manner may the angles *n* F *e*, *o* F *q*, be found, which angles are the equations to be ſubſtracted from the planet's place in its orbit, or to be added to it, to find the planet's place in its ecliptic: to be ſubſtracted, while the planet moves from the perihelion to the aphelion, and to be added, while it moves from the aphelion to the perihelion; the reaſon of which plainly appears from the inſpection of the ſcheme, the including triangle always being greater than the included. In the example before us, the table of equations of the planet's motion will be as follows:

°	′	°	′	″	°	′	″
22	30	13	07	30	2	05	38
45	00	22	30	00	1	47	30
67	30	28	07	30	0	36	54
90	00	30	00	00	0	00	00

The firſt column contains the planet's anomaly: the ſecond the equations for the true anomaly; which being found, the firſt column ſerves again for the true anomaly, and the ſecond column gives the equations for the planet's orbit: the third column contains the equation for reducing the planet's place in its orbit, to the planet's place in the ecliptic. In doing which, it is to be obſerved, that when the equation for the orbit is added, the equation for the ecliptic is to be ſubſtracted from the planet's place in its orbit: but when the equation for the place in the orbit is ſubſtracted, the equation for the ecliptic is to be added to the planet's place in its orbit. On theſe principles I have calculated the ſun's place for the year 1690, in ſeveral parts of every quadrant of the ecliptic, and found them agree with Mr. *Flamſteed*'s obſervations, ſo as to be generally within the extremes of his obſervations; which I think ſufficiently proves the accuracy of this method of calculating. I choſe the year 1690, becauſe his obſervations are moſt numerous for that year, and ſeem to be made with the greateſt accuracy.

29. Ever ſince *Kepler* diſcovered that the orbit of the planets is of an elliptic figure, moſt of the diſtinguiſhed aſtonomers have employed their thoughts in diſcovering various methods for forming equations, all or moſt of them from the properties of the ellipſis. Several of them are imperfect, and all of them ſeem difficult in practice. This method which I have given ſeems the moſt eaſy

to be comprehended. It is left to the practical aſtronomer to judge, whether it hath not likewiſe an advantage by its facility and accuracy in practice.

30. There is an apparent motion (not a real one) of the planets as well as of the fixed ſtars, when viewed from the earth, occaſioned by the progreſſive motion of light. This is ſo fully explained in the tranſactions of the *Royal Society*, by Dr. *Bradley*, the king's aſtronomer in the royal obſervatory, that it is needleſs to ſay any thing more particularly, than to appriſe the reader of it.

31. It was obſerved *, that the time of a planet's moving from the center of its centripetal and centrifugal motion, to either limit of that motion, is preciſely one quarter of the periodic time of its revolution in its orbit: therefore, if a planet moved round the ſun, without being under the influence of the action of any other body or planet, the places of the perihelion and aphelion would remain fixed. But if in its revolution, it paſſes near any other planet, ſuppoſe the other planet at P (fig. 11.) ſo as that it paſs the other planet in the firſt quadrant of its revolution, while it is acceding to the circle of its center of centripetal and centrifugal motion, the other planet at P by its reſiſting power, muſt leſſen the elaſticity of the æther, more on the ſide of the planet next the other planet, than on the ſide next the ſun; and therefore it muſt be impelled more ſwiftly by the rays of the ſun, towards the

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* Paragraph 17.

circle of the center of its centripetal and centrifugal motion, than it would have been had the other planet been abſent. In like manner, if it paſs the other planet while it is moving from the circle of the center to the limit of its centripetal motion, it will likewiſe move more ſwiftly to the limit, and will move ſome diſtance beyond the limit. If it paſs the other planet in the third quadrant, it muſt return more ſlowly to the circle in which the center of its centripetal and centrifugal motion moves; and if it paſs in the fourth quadrant, it will not reach the limit next the ſun, as well as move more ſlowly towards it. But if the other planet were betwixt the ſun and the planet, in any of theſe caſes, it would have the contrary effect, viz. of retarding the motion where it was in the other caſe accelerated, and of accelerating where in the other caſe it was retarded. The influence of any of the other planets on the earth is ſo ſmall, that it cannot be obſerved, except by the largeſt and niceſt inſtruments. For this reaſon we have not obſervations ſufficient for forming equations to determine theſe irregularities in the earth's motion, and indeed they are ſo ſmall, that they may be neglected. It is from this influence, that the place of the apſides of the earth's orbit is obſerved, after a number of years, to have moved forward. It is evident however, that this motion cannot be uniformly going on, in the ſame direction; but that at ſome times it may be direct, and at other times retrograde: in making obſervations, to determine the irregularities of this motion, the earth's paſſing of Mars and Venus will require the greateſt attention; for they muſt principally influence, in this caſe, when neareſt the earth. It may be obſerved, however, in general, that when the earth (for example) is between the circle of mean motion and the planet which influences its motion, the action of that planet on the earth is ſtronger, than when the circle of mean motion is between the earth and the planet; and therefore the effects of the planet's action muſt be greater in the firſt caſe than in the latter. But in the firſt caſe, the earth will run beyond the place, in the circle of mean motion, to which it would otherwiſe have come; as in the latter, it would have come ſooner to it: therefore the influence of the planets in general, muſt make the apſides of the earth's orbit move forward, or in the order of the following ſines. The like will happen to any other planet, under the like influence.

Now, I think, all the phænomena of the motion of the planets, hitherto obſerved by aſtronomers, may be clearly explained from the firſt principles of action in matter: that they are evidently the effects of the actions of reſiſting, moving, and elaſtic matter, as explained in the firſt chapter; and that theſe phænomena do not follow from gravitation alone, as explained by the followers of Sir *Iſaac Newton*,

Newton, and from the ſuppoſition of a certain degree of motion, at firſt communicated to the earth.

It remains only to ſhew, that the rotation of the planets round their axis, and the phænomena depending thereon, proceed from the ſame actions or cauſes. This will deſerve a particular chapter for the explanation of them. But before I proceed, I think it proper to draw ſome general corollaries, from what is obſerved of the motion of the planets in their orbits, which ſhall be the ſubject of the next chapter.

Yoursaccipiter

CHAP. IV.

Of the mean diſtances and excentricity of the orbits of the Planets and Comets.

1. I Have in the firſt chapter ſhewn, that it is probable that reſiſting matter, or bodies, may be divided into different ſpecies or ſorts, each differing in the degree of power eſſential to it. This chapter, in which the reaſon why the planets and comets perform their mean motions at ſeveral diſtances, and the reaſon of the different excentricity of their orbits, is inquired into, ſeems to be a proper place to diſcover, whether the diverſity of theſe phænomena, in one planet from another, but eſpecially the great differences between the phænomena in the planets and comets, in theſe reſpects, may not ariſe from the diverſity of the power of reſiſting, in the matter ſeverally compoſing them. For this purpoſe, I ſhall enquire in the firſt place, how theſe differences muſt be, ſuppoſing that all matter has the power of reſiſting equally and conſequently, that it is in the ſeveral planets and comets in the ratio of the quantity of matter in each. In the next place, I ſhall enquire what diverſity muſt happen in theſe phænomena, ſuppoſing the planets and comets to conſiſt ſeverally of matter, which in each has a different degree

of power of reſiſting; that by comparing the phænomena with the two ſeveral ſuppoſitions, it may appear which of them is true, by the agreement or diſagreement with the phænomena. But in the firſt place it is to be obſerved, that

2. The action of light is on the ſurface of bodies: but the action of the æther is on every part, or particle, which compoſe them.

3. I know of no method of diſcovering and determining the ratio of the force of light, to the gravitation of any determin'd quantity of matter; but it muſt be immenſely greater than that of any quantity of matter, which we can conceive; otherwiſe, it could not give that prodigious velocity to the great bodies of the planets, at ſo great a diſtance from the ſun, which it does. And if it be conſidered, that gravitation is cauſed by the differences only of the elaſticity of the æther, if there were not an immenſe difference between the force of light and theſe differences, there could be no proportion between the action of light and the elaſticity of the æther. But notwithſtanding of this, the ratio in which the force of light is leſſened, by the greater or leſſer force of gravitation, may be diſcovered. For, ſince the action of light is on the ſurface, and the gravitation is as the quantity of matter contained in that ſurface, the action of light muſt be always leſſened in the ratio of the quantity of matter to the ſurface.

4. Then if the quantity of matter be increaſed, ſuppoſing the ſurface remaining the ~~ſame~~, the diſtance from the ſun at which the action of light and gravitation become equal, will be reciprocal to the ſquare root of the quantity of matter: for gravitation, at the ſeveral diſtances, is in proportion to the quantity of matter, and the force of light increaſes reciprocally to the ſquares of the diſtances; and the greater the gravitation, the nearer to the ſun muſt it be equal to the force of light.

5. If the quantity of matter be the ſame, ~~but~~ the ſurfaces different, the diſtances of the equality of light and gravitation will be as the ſquare roots of the ſurfaces directly, or in ſpherical bodies, as the diameters.

6. Then the mean diſtances of the planets are in the ratio of their diameters directly, and of the ſquare root of the quantity of matter contain'd in them reciprocally. Therefore if the quantity of matter, in two or more bodies or planets, be in the ſame ratio to the ſquares of their diameters, or of their ſurfaces, they will move at the ſame diſtance.

7. Calling that proportion or quantity of matter, which at any determin'd diſtance is equal, in its gravitation, to the action of light, the ratio of equality; and ſuppoſing either the quantity of matter, or the ſurface, to be augmented above this proportion, then if the ſurface be augmented, the ſpherical body or planet will move farther from the ſun, in the increaſed ratio of the difference

between the ſquare root of the quantity of matter, and the ſquare root of the ſurface, which ſquare root is always in the ſame ratio with the diameter. But if the quantity of matter be increaſed beyond that proportion, then the ſpherical body will move nearer the ſun, in the increaſed ratio of the ſame differences; and at thoſe diſtances, the gravitation of that quantity of matter, and the ſurface, will be in the ſame proportion to each other, as they were at the place where they were before equal. This is the rule, which in the preceding chapter was promis'd to be given afterwards for diſcovering the place of equality, between the action of the æther and of light, or for placing the ſquare numbers, in paragraph 9 and 15 of that chapter; which ſquare numbers repreſent the action of light and gravitation at equal diſtances.

8. It was proved, in the preceding chapter, that the velocity with which a planet moves in its orbit, is reciprocal to the ſquare of the diſtance from the ſun; but that the velocity of a planet's centrifugal or centripetal motion, is reciprocal to the diſtance of the center of that motion from the ſun: then, if the quantity of matter in two ſeveral planets be the ſame, the diſtance to which they will run, by their centripetal or centrifugal motion, will be as their reſpective velocities, at the center of this motion. But if the quantity of matter in each be different, the diſtances will be as the velocities, multiplied into the quantity of matter: for the centripetal or centrifugal

motion of a planet, after the actions of light, and of the æther, become equal, is only continued by the reſiſting power in the planet, which at preſent is ſuppoſed always to be in the ratio of the quantity of matter. Thus, for example, ſecluding any conſideration of the atmoſphere, if a ball of twenty pounds weight was projected with the ſame velocity with a ball of ten pounds is, it muſt aſcend to twice the diſtance that the ball of ten pounds does.

9. Theſe things being premiſed, if the force of reſiſtance in all matter be the ſame, the excentricity of the planets muſt be reciprocal to the ſquares of their mean diſtances, multiplied into the quantity of matter contain'd in them. For ſince the velocity, with which the planets recede from their mean diſtance, is reciprocal to this diſtance, then making the mean diſtance *radius*, the excentricity muſt (where the quantity of matter is equal) be reciprocal to the ſquares of the *radii*; and when the quantity of matter is unequal, reciprocal to the ſquares of their mean diſtances, multiplied into the quantity of matter. To make this the more obvious, if the diſtances to which the planets run from their mean diſtance, either towards the ſun or from the ſun, were in all of them the ſame, then their excentricity would be reciprocal to their mean diſtance, or to the radius of their mean motion; but as the diſtances to which they recede from the mean diſtance is likewiſe reciprocal to their mean diſtance, their excentricity muſt

be reciprocal to the ſquares of their diſtances, that is to ſay, if the power of reſiſting be equal in all matter.

10. Now let us examine how far this agrees with the obſervations made of the excentricity of the planets. The mean diſtance of Mars is greater than that of the earth, and his diameter is leſs; then the quantity of matter in Mars muſt be much leſs than the quantity of matter in the earth, much leſs than in the ratio of their diameters. The excentricity of Mars, therefore, ought to be much leſs than that of the earth, both on account of the greater diſtance from the ſun, and the ſmaller quantity of matter: but in fact, the excentricity of Mars is much greater than that of the earth, and therefore this theory muſt be erroneous, and does not account for the motion of the planets, and it will much leſs account for the excentricity of the comets.

11. Let us next examine, whether this remarkable phænomenon of the excentricity of the planets and comets can be accounted for, by ſuppoſing, as was propoſed to be done, that the ſame quantity of matter, in different bodies, may have different degrees of the power of reſiſting. If by this ſuppoſition only, this abſtruſe phænomenon can be well and clearly ſolved, it will give the ſtrongeſt probability that in fact it is ſo, and therefore deſerves conſideration.

12. For which purpoſe it is to be obſerved, that, ſuppoſing reſiſting matter to be of different ſpecies, one having a greater power of reſiſtance than another, the gravi-

tation of any body towards any one other certain body, will not in any caſe be thereby affected; but that the gravitation of any body towards another will, at all diſtances, be as the quantity of matter contained in it, without any regard to that quantity of matter having a greater or leſſer power of reſiſting. Fer gravitation is cauſed by the difference of the elaſticity of the æther, greater at a greater diſtance, and leſs at a leſs diſtance: but ſuppoſing the power of any body, which gravitates towards another, to be of any degree of reſiſtance either great or ſmall; this can in no manner alter the gravitation, by making it either greater or leſſer. For though the reſiſtance in the gravitating body leſſens the elaſticity of the æther all round it, equally at equal diſtances, yet the differences of the elaſticity of the æther at the ſeveral diſtances from the body, toward which the other gravitates, will remain the ſame; becauſe, if from unequal quantities, equal quantities be ſubſtracted, the differences of the unequal quantities will remain the ſame after ſubſtraction as before.

13. Again, the reſiſting power of a planet, be it greater or leſſer, no way affects the mean diſtance of that planet's revolution in its orbit round the ſun; but, that this mean diſtance will be always in the ſame ratio of the action of light on the ſurface to the quantity of matter contained in it, whatever degree of reſiſtance that quantity of matter hath. That this may the more eaſily be underſtood, let it be ſuppoſed, that all actions ceaſe except the action of

light, and the action of reſiſting in any quantity of reſiſting matter; then, tho' the action of reſiſting be negative to motion, yet by the nature of this action, whereby it oppoſes any change in its preſent condition, it at the ſame time preſerves any other action communicated to it. It is to be obſerved likewiſe, that the action of light is renewed every inſtant, by the reiterated impreſſions from new emiſſions of light, and that the reſiſting power preſerves all theſe new actions. Then ſuppoſe the ratio of the action of light at every inſtant to be equal to + 20, and the ratio of a certain quantity of reſiſting matter equal to — 100. In the firſt inſtant, the action of motion communicated to the reſiſting quantity of matter and preſerved by it is — 100 + 20 = — 80; in the ſecond inſtant, — 80 + 20 = — 60; in the third inſtant, — 60 + 20 = — 40; in the fourth inſtant, — 40 + 20 = — 20; in the fifth inſtant, — 20 + 20 = 0; that is, at this time the action of the reſiſting matter and of light are equal, and being negative to each other, their joint action is 0; then in the ſixth inſtant the motion of the reſiſting body muſt become + 20, becauſe the action of the reſiſting power is intirely counter-ballanced by the preceding actions of light; in the ſeventh inſtant, + 20 + 20 = 40, *&c.* and ſo every inſtant continually increaſes. Or theſe contrary actions may be expreſſed by the following ſeries of numbers — 100 + 20, — 100 + 20 + 20, — 100 + 20 + 20 + 20, — 100 + 80, — 100 + 100, — 100 + 120, — 100 + 140, *&c.* continually increaſing, till the

motion

motion of the reſiſting body come to that degree of motion or velocity, that there is no difference between the degree of velocity in the reſiſting body, and the degree of velocity in light. This, from the nature of the actions of light and reſiſtance, appears evidently to be the caſe, when all other actions are ſuppoſed to ceaſe. Then any quantity of reſiſting matter may receive any degree of velocity, at any diſtance from the ſun, if nothing elſe but its own reſiſtance were to hinder or obſtruct.

14. In the preceding paragraph it is ſuppoſed, that the ratio of the action of light, to the ratio of the action of the reſiſting body, continues the ſame every inſtant, that the action of light is impreſs'd: but, if the ratio of the action of light decreaſe every inſtant, as the ſquares of decreaſing arithmetical proportionals, then the ratio of the velocity acquired by an infinite number of the actions of light decreaſing in this manner, will be one third of the velocity which would be acquired by an infinite number of actions, all equal to the firſt or greateſt action in the decreaſing infinite number. And the velocities acquired by an infinite number of equal actions, which are of different ratios to the action of reſiſtance, will be in the ratio of the actions. In the ſame manner, and for the ſame reaſon, that a cone compoſed of an infinite number of ſurfaces equal to the ſquares of arithmetical proportionals, is one third of a cylinder of the ſame height and baſe, and that cylinders of the ſame height are in the ratio of their baſes; the baſes repreſenting the velocity,

and the height the reſiſtance; the *momentum* of the planets is in the compound ratio of the velocities and reſiſtance. But whatever the reſiſtance be, the velocity at equal diſtances from the ſun is the ſame. For whatever be the ratio of the action of reſiſtance to one ſingle action, or emiſſion of light, unleſs it be infinitely greater, it cannot be equal to an infinite number of thoſe actions: and if theſe actions continually decreaſe in any ratio of the diſtance, the reſiſtance can only be equal to it at an infinite diſtance.

15. If the differences of two or more quantities bear no proportion to another quantity, or if the ratio of theſe differences to that other quantity be infinitely ſmall, or leſs than any that can be determined, theſe quantities, tho' really different, will all of them have the ſame finite ratio or proportion to that other quantity: for on this maxim the arithmetic of infinites is founded. But the differences of the reſiſtances of ſeveral finite quantities, bear no proportion to the infinitely repeated actions of light, and therefore they all acquire the ſame velocity, at equal diſtances from the ſun. Notwithſtanding this, their *momentums* will be different, in the ratio of the differences of their ſeveral quantities of reſiſtance: becauſe one finite multiplied into an infinite, has the ſame ratio to another finite multiplied into the ſame infinite, that the one finite has to the other finite.

16. But the action of the æther, by which gravitation is cauſed, is repeated every inſtant, alternately with the

action of light; and contrary to the action of light in the direction from the ſun. Were the action of the æther every where equal to the action of light, it muſt every where obſtruct the motion of any reſiſting body, in a direction from the ſun. But as the action of light is greater than the action of the æther while nearer the ſun, but decreaſes faſter at equal diſtances from the ſun, than the action of æther, this at laſt muſt ſomewhere become equal to the action of light: and the reſiſting body can recede no farther from the ſun, by the action of light only, than to the place of this equality.

17. It follows then, than the diſtance of the mean motion of the planets does not depend on the quantity of reſiſtance, but on the quantity or force of gravitation in them, which is always in proportion to the quantity of matter, and not as the quantity of reſiſtance in them.

18. But it is the reſiſting power only in the planets, which gives and continues their centripetal and centrifugal motion, after the actions of the æther, or gravitation, and of light, become equal. Therefore the diſtance, to which this centripetal or centrifugal motion, in the planets, is carried (other things being alike) muſt always be in proportion to the degree of reſiſtance. Thus by increaſing the quantity of reſiſtance, without increaſing the quantity of matter, the planets and comets may have any excentricity to any mean diſtance that can be ſuppoſed.

19. If then the diameter and mean diſtance of a planet be given, the ratio of the quantity of matter contained in it, to the quantity of matter contained in any other planet, whoſe mean diſtance and diameter are likewiſe given, may be found. And if the ratio of the quantity of matter and excentricity of one planet or comet to the quantity of matter and excentricity of another be given, the ratio of the quantity or degree of reſiſtance in the one to the other may be found.

20. Since the apparent attraction or gravitation towards any body is not cauſed by the quantity of matter in that body, but by the quantity or degree of reſiſtance in it, there may be a greater apparent attraction towards a ſmaller quantity of matter.

21. From the diameters and mean diſtances of Mars and of the earth, the quantity of matter in Mars muſt be much leſs than the quantity in the earth. But from their excentricities the quantity of reſiſtance muſt be much greater in Mars than in the earth. Therefore the motion of the earth in its orbit, muſt be more diſturbed by the apparent attraction of Mars, than the motion of Mars by the attraction of the earth.

22. The quantity of reſiſtance, and conſequently of attraction in the comets, in proportion to the quantity of matter in them, muſt be vaſtly greater than in any of the planets.

23. It ſeems probable, that the quantities of atmoſphere ſurrounding the planets or comets are in the ratio of their reſiſtance. This appears from the large atmoſphere accompanying the comets: and the light which Mars reflects being more obſcure than in the other planets, it ſeems probable that Mars has a greater atmoſphere, in proportion to his bulk, than the others.

24. By the atmoſphere's being much rarer or leſs denſe than the body of a planet, ſuch planets as have the largeſt atmoſphere, muſt have their mean diſtances farther from the ſun, than otherwiſe would have happened from the quantity of matter. For thereby the ſurface is much enlarged, in proportion to the quantity of matter contained in it.

CHAP. V.

The rotation of the Sun and Planets round their Axes, the inclination of their Axes to the Ecliptic, and the phænomena thereon depending.

1. THE ſun is obſerved to turn round its axis in about 27 days, and it is obſerved, that the ſection of every planet through its center, by a plane perpendicular to its axis, is larger, or has a larger diameter than any other ſection of the planet. The circle terminating this ſection is commonly called the planet's equator. The axis is likewiſe obſerved to be the ſhorteſt diameter of the planet. It ſeems then probable, that the ſun's longeſt diameter is in the plane of his equator. I know not whether this difference of the ſun's diameters has been obſerved; but it is poſſible that the different poſition of its equator to the earth, at different times, may be the reaſon why ſeveral aſtronomers have aſſigned different lengths to the ſun's greateſt and leaſt diameter.

2. As the directions of the action of light from the ſun, of the action of its reſiſtance, and of the action of the æther, are all in lines every where from and towards the ſun's center; there is nothing in the direction of theſe

actions that can give the ſun a rotation round its axis. However, if it be conſidered, that the light which is perpetually emitted from the ſun, muſt previouſly have been intermixed with the reſiſting matter of the ſun's body, and of its atmoſphere; and as the action of motion eſſential to light, is in all directions equal, it muſt continually exert that action; but the determination of that motion in any one direction, is by the reſiſtance being leſs in that direction than in any other: ſo that though the action of light on the ſurface be in all directions equal, yet within the ſurface (ſuppoſing the ſun in abſolute reſt) this motion muſt be in directions directly from the center, becauſe in all other directions the reſiſtance is greater, and equal on all ſides equidiſtant from the center. But if by any means, the reſiſtance on one ſide be leſſened more than in the oppoſite ſide, the action of light or motion will be communicated more on that ſide than on the other, and thereby a rotation enſue. When once this motion is begun it muſt always be continued, while the action of light within the ſurface continues; becauſe the oppoſition to the action of light, in the direction in which this motion is perform'd, is leſs than in any other.

3. The firſt beginning of the ſun's rotation may be accounted for as follows, viz. Suppoſe S the ſun's body, P a planet moving round the ſun, ſuppoſe Mercury, let S P (fig. 13.) be a line connecting the centers of the ſun and planet. Then the elaſticity of the æther contained within

the body of the ſun, will be moſt leſſened in the lines connecting theſe centers, and on each ſide of it, the æther's elaſticity increaſes in ſome ratio of the diſtance from it; conſequently the reaction of motion from light, within the hemiſphere A B of the body of the ſun, will be towards that line on each ſide of it, and if nothing were to hinder, the reſiſting matter on each ſide of this line would preſerve this action or tendency to motion, which is commuuicated to it by the æther, as in gravitation. Suppoſe that the planet in the next inſtant of time was moved to *p*, then the elaſticity of the æther will, in like manner, be leſſened on each ſide of the line S P, and on the reaction of the æther, the motion within the body will be directed towards the line S *p*; therefore the motion, which was preſerved in the parts of the body contained between the lines S P and S *p* will have their determination or direction of motion changed, from being towards S P, and be towards S *p*; but there will be no change in the direction of the motion preſerved by the reſiſting matter in the ſide A C of S P; conſequently, when the planet comes to *p*, there will be a greater quantity of matter having motion in the circular direction from A towards D, than in the direction from D towards A. This difference will continually increaſe, as the planet moves forward, till it come to the direction of the point E oppoſite to P: at which time the quantity of matter, having a circular direction of motion from A towards B D E, will be three ſourths, while the quantity with the oppoſite direction from A to E, is one fourth of the whole: then the body of the ſun will by this means acquire a rotation round its axis, and having once gained a rotation it muſt be always continued.

4. For a ſimilar reaſon the general current of the tides are from weſt to eaſt, and thereby ſhips in the *Atlantic Ocean* make more ſpeedy voyages from *North America* to *Europe*, than from *Europe* to *North America*; and ſhips commonly are a-head of their reckoning in ſailing from *America* to *Europe*, while the reverſe happens in ſailing from *Europe* to *America*.

5. The cauſe of the rotation of the planets round their axes, and which move round the ſun from weſt to eaſt, is generally and eaſily explained thus: The parts of the planet fartheſt diſtant from the ſun have a greater degree of motion or velocity from weſt to eaſt, than the parts nearer the ſun; conſequently, there will be leſs reſiſtance to the action of light, contained within the ſurface of the planet in the direction which moſt coincides with the direction of the motion of the planet in its orbit, than in that direction which is oppoſite to the direction of this motion; therefore the direction of the action of light contained within the planet's ſurface, is from weſt to eaſt.

6. That this rotation of the planets is really produced by the action of light within the planet's body and atmoſphere (ſince all motion is produced by light) ſeems to be con-

firmed from this conſideration, that their atmoſphere in the rotation keeps equal pace with the ſolid parts of the planets in this rotation: For, if this motion were impreſs'd by any thing external on the ſurface, it being a fluid could not communicate this motion to the ſolid parts, no more than the tides in the ſea communicate their motion to the land; at leaſt the atmoſphere could not communicate an equal degree of motion: for if the motion were impreſs'd on the ſolid parts of the planet, the ſolid parts would not communicate an equal degree to the atmoſphere, ſince they are not in coheſion with each other.

7. This is further confirm'd by conſidering, that as four ſatellites or moons move round Jupiter, there muſt be a very conſiderable emiſſion of light from Jupiter, to give them motion, more than can be ſuppoſed from the reflection of the ſun's light only, which likewiſe ſeems to be manifeſt from the great ſplendor of Jupiter at ſo great a diſtance from the ſun. Now, ſuppoſing that Jupiter contains a greater quantity of light than any of the inferior planets, the reaſon of his ſwift rotation round his axis may be accounted for from thence, and which it will be difficult to ſhew how it can happen from any external application that can be ſuppoſed. It ſeems likewiſe probable that the reaſon of the moon's having no rotation, is from its emitting a very ſmall quantity of light, and for that reaſon the longeſt diameter is always directed to the earth,

the natural effect of its gravitation, where no other action hinders.

8. It is not only probable that the earth emits light, but it is neceſſary for the motion of the moon in its orbit round the earth; and is farther confirmed by that part of the body of the new moon being ſeen, which is not illuminated by the ſun, and by the whole body's being ſeen in a total eclipſe of the moon: both which may be by the reflection of light emitted from the earth. The part of the moon, which is not illuminated in the new moon, appears more diſtinct or more illuminated, than the body of the moon in the middle of a total eclipſe, becauſe the light from the earth to the moon is increaſed by the reflected light from the ſun, which cannot be in a total eclipſe of the moon. The part not illuminated by the ſun cannot be ſeen, while any part of the moon's body is illuminated by the ſun, the leſſer light being obſcured by the greater, as the fixed ſtars are in day-light by the ſun. This probably is the reaſon, why the obſcured part of the moon is not ſeen in a partial eclipſe.

9. From what precedes it follows, firſt, that (other things being alike) the rotation of any planet muſt be ſwifter as its motion in its orbit is. 2dly, That the larger the planet's diameter the ſwifter the motion, becauſe the greater difference between the fartheſt diſtant parts and neareſt to the ſun. 3dly, The greater emiſſion of light from any planet, the ſwifter its rotation.

10. If a body perfectly ſpherical be placed in the æther within the ſphere of action of the light of the ſun, and of the reſiſting matter contained in it, there can be no difference of the action and reſiſtance of the æther on this body: but, if it be not perfectly ſpherical, ſo that one or more ſections of it, by planes through its center, be greater than others, as it muſt be in a ſpheroid, the action of light and reaction of the æther will have different effects upon it, when it is in different poſitions.

11. The equatorial diameter of the earth (for example) is longer than its axis, and the equatorial plane is larger than any other ſection of the earth. 2. The ſection of the earth through the poles of the equator is the leaſt. 3. The illuminated hemiſphere of the earth, in the ſolſtice, makes nearly an angle of 23 ½ degrees with the plane of the equator. 4. In the equinoxes the illuminated hemiſphere is at right angles with the equatorial plane. 5. The inclination of the illuminated hemiſphere to the equatorial plane continually increaſes from the equinox to the ſolſtice. Then the force or action of light on the illuminated plane is greater in the ſolſtices than in any other place of the earth's orbit, and is leaſt in the equinoxes. The reaction of the æther, or the force of gravitation, on the oppoſite ſide of the equatorial plane, is in like manner greateſt and leaſt, and increaſes or decreaſes, in the ſame ratio the force of light does, in theſe different places of the earth's orbit.

For the direction of both their actions are in lines paſſing through the ſun's center, and both increaſe reciprocally to the ſquares of the diſtances from the ſun.

12. The neareſt extremity to the ſun of any line or ſection, or plane of a planet, that is inclined to the plane of its orbit, gravitates more towards the plane of its orbit than the fartheſt diſtant extremity of that line, ſection, or plane; and ſuppoſing that the planet was to turn upon an axis inclined to the line connecting the centers of the ſun and planet, and in the plane of the orbit, the plane of the equator, and all its diameters or lines paſſing through it, muſt be likewiſe inclined, and the gravitations of the two extremities of the inclined lines or plane, muſt act in oppoſition to each other, the gravitation of the neareſt extremity drawing that extremity towards the plane, while the gravitation of the other extremity draws the neareſt extremity from it. But the action of light in the extremity of the ſame inclined line or plane, produces a contrary effect, by its impelling the neareſt extremity of the ſame inclined line from the plane of the planet's orbit, and by impelling the fartheſt diſtant extremity towards the plane of the orbit. Then the force by which the inclined plane gravitates towards the plane of the orbit, is the difference between the gravitation of the two extremities; and the force, by which the inclined plane is impelled from the plane of the orbit, is the difference of the action of light on the two extremities. But ſince the action or force of

light in the ſame diſtance, decreaſes faſter than the force of gravitation, the difference between the action of light on the two extremities, muſt be greater than the difference of gravitation on the ſame extremities; therefore the inclined plane muſt always be impelled from the plane of the orbit, and the force by which it is impelled will be as the differences of theſe differences. As the angle which the inclined plane makes with the plane of the orbit becomes greater, the differences of the force of light and gravitation on the two extremities will continually grow leſs, ſo that when the inclined plane comes to a right angle with the plane of the ecliptic or orbit, the difference is = 0: therefore the reſiſtance of the planet's body will become equal to the force by which the inclined plane is impelled from the plane of the orbit, ſomewhere before this plane can be at right angles to the plane of the orbit.

13. While the plane of the equator is in the plane of the orbit of a planet, the plane of the equator cannot by the gravitation in the planet itſelf, or the action of light, be moved out of the plane; but if by the apparent attraction of another planet, or of the moon, or of a ſatellite, placed out of that plane, the equator once become inclined to the plane of the orbit, then the action of light may impel it ſtill farther, ſo that it may make a greater angle with the plane of the orbit, till the reſiſtance of the

planet becomes equal to the impelling force of light, which, as before obſerved, grows leſs as the angle grows greater.

14. The reſiſtance which a planet has, whereby it oppoſes any change in the poſition of its axis, is not only the reſiſtance, properly ſpeaking, of the reſiſting matter, which has the ſame force at reſt as in motion, but the *momentum* of a planet turning round its axis, with a degree of velocity, and is as the velocity multiplied into the reſiſting force of the planet. This *momentum*, or reſiſtance, is ſo very great, in proportion to the differences of the differences of gravitation and of light, that the poſition of the axis of the planet muſt remain the ſame, except where a planet's orbit has a very great excentricity. For example, the difference of the diſtance of the earth, in its perihelion and aphelion, is ſo ſmall, that though from theory, the inclination or angle, which the earth's axis makes with the plane of the orbit, muſt be leſs in the aphelion than in the perihelion, becauſe the ratio of the gravitation to the action of light is greater in the aphelion, yet this difference of differences is ſo ſmall, in reſpect to the velocity with which the earth turns round its axis, multiplied into its reſiſtance, that it is no wonder that it cannot be diſcovered by the moſt accurate inſtruments. The great velocity of Jupiter round his axis, probably, is the reaſon why the plane of its equator is very little moved

from the plane of its orbit by his four ſatellites, though the action of the moon, on the plane of the earth's equator, has become ſenſible to Dr. *Bradley*'s accurate obſervations, and which had eſcaped the obſervations of aſtronomers before him.

15. Sir *Iſaac Newton* was the firſt who explained the cauſe of the preceſſion of the equinoxes, but his reaſoning on that head is not eaſily comprehended. I ſhall next attempt to ſhew how it is a neceſſary conſequence of the principles of action in matter, by which the planets are put in motion, and their motion regulated. Let S be the ſun (fig. 14.) ACDFGHIK the orbit of the earth, or the ecliptic, A S G the ſolſtitial colure, D S I the equinoctial, Then, when the earth is in the ſolſtitial colure, the axis of the earth is in the plane of the colure, and inclined to the axis of the orbit, or of the ecliptic, ſo as to make an angle nearly of 23 $\frac{1}{2}$ degrees, and the plane of the earth's equator, is erect or perpendicular to the plane of the ſolſtitial colure, or to a plane paſſing through and parallel to the axis of the ecliptic. Then all the lines in the equinoctial plane, parallel to the plane of the orbit, are biſſected by the ſolſtitial colure, and all the parts of theſe lines in the equatorial plane, on each ſide, equidiſtant from the ſolſtitial colure, are likewiſe equidiſtant from the ſun. In this caſe the equatorial plane of the earth cannot, by any action of light, be more inclined to the ſolſtitial colure on the one ſide than on the other; and if the earth, in

moving from weſt to eaſt in its orbit, turn its equatorial plane in ſuch manner, that it be always perpendicular to a plane, parallel to and paſſing through the axis of its orbit, as it muſt do by the action of light, if nothing elſe hinder, the axis of the earth muſt always make an angle of 23 $\frac{1}{2}$ degrees on all ſides of the axis of the orbit; for in the preſent caſe I ſuppoſe that the poſition of the earth's axis remains otherwiſe fixed, or whatever force there is in the action of light to change it, it is exactly counterballanced every where by ſome other power, that is to ſay, that the direction of the earth's axis in the quadratures will be in the line E Q, and in the octants in the line B *b*. If this were the caſe, and nothing propoſed to alter the inclination of the axis of the earth to the axis of the ecliptic, the poles of the earth in the courſe of one year would deſcribe a circle round the poles of the ecliptic at 23 $\frac{1}{2}$ degrees diſtance from it. For as the diameter of the ecliptic is but as a point to the diſtance of the ſtars from the earth, this circular motion of the poles of the earth will be the ſame in appearance, as if the center of the earth were in the center of the ecliptic, and its axis continued to the fixed ſtars by its revolution, deſcribed a conical ſurface, whoſe apex is in the center of the ecliptic and its baſe a circle of 23 $\frac{1}{2}$ degrees radius, round the poles of the ecliptic.

16. It is ſuppoſed in the preceding paragraph, that there is nothing to hinder the action of light in making

the changes in the poſition of the equatorial plane of the earth, which muſt enſue from that action alone; but the caſe is far otherwiſe: for the *momentum* of the earth in its rotation round its axis, or the velocity of the earth in its rotation, multiplied into its power of reſiſtance, oppoſes any alteration of the earth's equatorial plane; and, as the force by which light changes the ſituation of that plane, is only the difference of the force of its action on the parts of that plane, on thoſe that are neareſt and moſt diſtant from the ſun, leſs than the difference of gravitation in the ſame parts, the ratio of this action muſt be very ſmall, in proportion to the force which oppoſes any alteration in the ſituation of the earth's equatorial plane. Yet though it be exceeding ſmall, it is diſcovered by the accurate obſervations of aſtronomers, to cauſe a preceſſion of the equinoxes of about fifty ſeconds in a year. The cauſe of which I propoſe next to explain.

17. When the earth has moved from the ſolſtice at A, to the octant at C, ſuppoſing that the equatorial plane preſerved its ſituation by the *momentum* of its rotation, then by the ſtronger action of light on the nearer parts to the ſun at E, theſe parts will recede from the ſun, and thereby the pole of the earth will receive a motion following the order of the ſigns. When the earth comes to the quadrature at D, the equinoctial plane will be in the direction of the rays of light, and they can have no force to move it to either ſide. When it comes to the octant at F, the

neareſt parts to the ſun at E will recede from the ſun, and thereby the pole receive a motion contrary to the order of the ſigns. When it comes to the oppoſite ſolſtice at G, the parts on each ſide of the plane of the ſolſtitial colure are equidiſtant from the ſun, and therefore the action of light can give no angular motion to it. When the earth comes to the third octant at H, the parts neareſt the ſun at Q will recede from the ſun, and the pole of the earth again receive a motion following the order of the ſigns. In the quadrature at I the pole can receive no motion, and at the laſt octant at K, the pole will again receive a motion contrary to the order of the ſigns. So far conſidered, the pole of the earth in its revolution in its orbit, will twice receive an angular motion in the order of the ſigns, and twice contrary to that order; but while it receives motion in the order of the ſigns, that motion concurring with the earth's rotation, the rotation will be increaſed by it, and conſequently leſs force is applied towards the alteration of the ſituation of the equatorial plane: and in the other caſe, when the action of light is contrary to the rotation, a greater force is applied to the plane of the equator, and thereby the motion of the pole contrary to the order of the ſigns will be greater than that in the ſame order.

18. I cannot conceive how any alteration of the inclination of the axis of the earth, ſuppoſing no other alteration in the ſituation of the plane of the equator, can produce any alteration in the place of the equinoxes; becauſe the

motion of the pole muſt in that caſe be in the ſolſtitial colure. For a meer nutation of the axis, without any angular motion, tho' it produce a ſenſible alteration in the declination of the ſtars, yet they may preſerve the ſame diſtance from the equinoctial points: therefore ſince Dr. *Bradley* * has likewiſe obſerved a difference in their right aſcenſion, this muſt prove that the axis of the earth has not only a nutation but an angular motion likewiſe, and that as the greateſt differences of the nutation is when the aſcending node of the moon's orbit is in the equinoctial point, ſo the greateſt difference in the angular motion muſt be when the ſame node is in the ſolſtices. If theſe differences be exactly equal, then the pole of the earth deſcribes a circle of eighteen ſeconds diameter, as Dr. *Bradley* ſuppoſes; if unequal, it muſt deſcribe an ellipſis: ſubject however to many variations, not only according to the diſtance of the moon from the earth, but likewiſe the earth's diſtance from the ſun, and this not only on account of the different apparent attractions, but likewiſe the different force of light. Though theſe differences muſt be ſo ſmall as to be of little uſe, and may eſcape the obſervations of the moſt accurate aſtronomers with the beſt inſtruments, yet I have been thus particular in obſerving upon them, becauſe they lead us to conſider what effects the ſame cauſes may have on the moon's motion in its orbit, where they may be very conſiderable.

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* Philoſoph. Tranſ. *January* 1747-8. N° 485.

19. The accuracy of the ingenious Dr. *Bradley*'s obſervations gives hopes, that he may confirm what I ſuggeſt of the differences of velocity of the earth's rotation, in the ſeveral octants of its orbit, and thereby ſhew the neceſſity of a new equation of time. This may be done by obſerving the difference of the ſiderial day, when the earth is in the octants, or by obſerving the tranſit of two ſtars. By obſerving the tranſit of two ſtars likewiſe may be diſcovered, whether a pendulum clock be not affected by the different poſitions of the moon, from the ſame cauſe the tides are.

20. The only general phænomenon in the motion of the planets which remains to be explained, is to ſhew from the ſame principles, why the planets move within that portion of the heavens called the zodiac. For this purpoſe it is to be obſerved, that the rotation of the ſun round its axis proves, that its equatorial diameter is longer than its axis, not only becauſe it is ſo in all the planets, but likewiſe becauſe, for this reaſon only, the rotation is always continued in the ſame plane; and any action applied to the extremity of the longeſt diameter, muſt give a rotation in the plane of that diameter, ſuppoſing the ſame action were equally applied to the others. For this reaſon it is wrong to ſuppoſe, that the ſpheroidal ſhape of the ſun and planets is cauſed by their rotation: for if it were ſo, this ſpheroidal ſhape muſt continually increaſe, contrary to all obſervation. If the equatorial diameter of the earth increaſed, the obliquity of the ecliptic muſt likewiſe

increaſe; the contrary of which ſeems true, by comparing the obſervations of the ancients with the obſervations of this age.

I know not, that this difference in the ſun's diameter has been obſerved by aſtronomers, ſeveral reaſons may be given why it ſhould eſcape their notice. But it is poſſible, that the different poſitions of the ſun's equator, at different times and in different places, may be the reaſon why different aſtronomers have aſſigned different lengths to the ſun's greateſt and leaſt diameter.

21. If then the plane of the ſun's equatorial place be extended infinitely, the elaſticity of the æther in that plane muſt, every where, be more leſſened than in any other place equidiſtant from the ſun's center: and becauſe of the ſpheroidal figure of the ſun, the elaſticity of the æther muſt, at equal diſtances from the center, be gradually more and more leſſened from the axis indefinitely extended, to this plane of the equator. But this difference, at equal diſtances from the ſun in the ſame parallel, grows leſs as the diſtance from the ſun's center increaſes, for a double reaſon, *viz.* by the arches increaſing as the diſtance from the ſun, and the difference of elaſticity likewiſe decreaſing as the ſquares of the diſtances from the ſun. Then any body moving round the ſun will be impelled towards this plane; and the nearer to the ſun they move, with the greater force will they be impelled towards the place of the equator thus extended.

22. For this reaſon Mercury moves nearly in the plane of the ſun's equator; and Venus, which is at the next neareſt diſtance, deviates leſs from this plane than any of the other planets which are more remote. The inclination of the plane of a planet's equator to the plane of the ſun's equator, is the cauſe of its deviating from the plane of the ſun's equator: as the axis of a bias bowl, being inclined to the plane on which it rolls, is the cauſe why it deviates from the direction in which its motion was impreſs'd. The axis of the earth is the moſt inclined of any of the primary planets; and therefore the plane of its orbit deviates the moſt of any of the primary planets from the plane of the ſun's equator.

23. The mean diſtance of the comets from the ſun is ſo very great, that the difference of the elaſticity of the æther at that diſtance, in the plane of the ſun's equator and in other places equidiſtant, muſt be exceeding ſmall; and therefore the comets move indifferently in all directions.

CHAP. VI.

Of ſome other Phænomena abſtracted from the Motion of the Planets.

1. I Once had thoughts of attempting an explication of the phænomena in Sir *Iſaac Newton*'s Optics, as a farther Proof of the doctrine which I attempt to teach. I ſet my thoughts on them, becauſe of the accuracy of his experiments, which is abſolutely neceſſary to eſtabliſh my theory. But I now doubt much of my having either leiſure or length of life, ſufficient to purſue ſuch kind of ſpeculations, with that accuracy which ſuch a deſign requires. I expect that a general conſideration of ſome other phænomena may ſerve to confirm my opinion, that I have truly ſtated the firſt cauſes of action in matter. Therefore I ſhall briefly hint at the explication of ſome phænomena, leaving the more particular proſecution of ſuch like ſpeculations to others, who have more life and more leiſure to purſue them.

2. From what precedes, it ſeems evident that the ſun and planets are balls of reſiſting matter, ſurrounded by an atmoſphere emitting rays of light: and that probably the greater the power of reſiſting in the matter which forms

theſe balls, the greater is the atmoſphere which ſurrounds them, and the more denſe it is. This appears moſt evident in the comets; but it ſeems probable that the atmoſphere of the ſun is the greateſt and moſt denſe of all, as the reſiſting power in the ſun is; otherwiſe the great quantity of light, which it continually emits, muſt break the coheſion of its parts, nor could it have ſuch a ſtrong apparent attraction as it evidently has.

3. The atmoſphere or air ſurrounding the planets and comets, ſeems to conſiſt of the ſmalleſt parts of reſiſting matter, ſeparated from their coheſion with each other, and from the body of the planets, and emitting plenty of light in all directions. That the great quantity of light, which they emit, is ſuperior to their mutual apparent attraction, and thereby they are impelled from each other, or ſeem to fly from each other. This ſeems to be the true cauſe of what is called the elaſticity of the air; and this emiſſion of light in greater quantity, is the reaſon that air ſeems to be neceſſary in all fermentations. Air is not only a neceſſary aſſiſtant to all fermentations, but is likewiſe generated from them.

4. By theſe means the atmoſphere is in a continual fermentation, or inteſtine motion of its parts. Fire and flame are the moſt violent fermentations, fire in the ſurfaces of ſolids, and flame in the vapours or atmoſphere emitted from them. By whatever means the ſmalleſt parts of bodies are ſeparated, they then emit

light, which before was retained by their united and ſtronger apparent attraction, and begin to ferment; fermentation beginning always on the ſurfaces ſeems to confirm this.

5. Solid bodies have ſome degree of fermentation, tho' in many it be exceeding weak, becauſe we find in time the coheſion of their parts change, they grow lighter, many of their parts ſeparate, and ſome bodies become, in their texture, ſomething intirely different from what they were: as the properties of old cheeſe are intirely different from thoſe of new, or of curd, wine from vinegar, *&c.* As this cannot be done without motion, ſolid bodies muſt contain and emit light. The electrical experiments fully confirm this.

6. The difference between electric and non-electric bodies may ariſe from the great difference in the quantity of light which they emit, and light's being more eaſily ſeparated from the one than the other. The electrics emitting a great quantity of light, and the non-electrics a ſmall quantity, and apparently attracting light with greater force. Perhaps it may be of further uſe, in conſidering the phænomena of electricity, and in any attempt to explain them, that it ſeems probable that the ſmall bodies as well as the great are ſurrounded with an atmoſphere peculiar to themſelves, and which in contiguous bodies unite. That the electrics have a much longer atmoſphere, and more repleniſhed with light, than the non-electrics have. The

friction cauſes a great emiſſion of light, and the uniting the atmoſphere of an electric with the atmoſphere of a non-electric, occaſions the exploſion without any actual touch between the bodies; for the approach of a body, with a great apparent attraction, may greatly leſſen or intirely deſtroy any attraction towards the other. If the atmoſphere of a non-electric be deeply charged with light in ſmall quantities at any time, from an electric, excited by friction to emit its light, this non-electric becomes thereby an electric. By ſomething of this kind the different apparent attraction, and conſequently the different power of reſiſting, in ſmall bodies, may be diſcovered.

7. Fluid bodies contain perhaps more light than the ſolid in proportion to their reſiſting power; and this may be the cauſe of their fluidity. For generally heat increaſes and cold leſſens fluidity.

8. The ſenſe of heat ſeems to be produced by the repeated vibrations of the ſmall parts of bodies, agitated by light; for pure light ſeems to produce no ſenſe of heat. This is confirmed by the ſenſe of heat being no where in proportion to the quantity of light, but in proportion to the vapours which ſurround our bodies. For this reaſon the moſt ſultry days are when the air is fulleſt of vapours; for this reaſon, the vallies are the moſt ſultry, and the higheſt mountains, even in the torrid zone, are extremely cold.

9. Cold, ſimply taken, ſeems to ariſe from the action of reſiſting matter, communicated to our ſenſe by the vibrations of the æther: but our ſenſe of heat and cold really ariſe from the complicated actions of light and reſiſting matter; for in no place can theſe actions be communicated to our ſenſes ſeparately and ſingly. We judge of heat and cold, according to the degrees of the action of light or of reſiſting matter, in the complicated action.

10. The force of fire is the *momentum* ariſing from the united actions of motion and reſiſtance: therefore the parts of bodies, which have the greateſt reſiſtance, agitated with the ſame degree of motion, burn with the greateſt violence.

11. It may be worth an enquiry, whether the diſtinction of acid and alkali, do not ariſe from a like compoſition of the actions of reſiſting matter and light; when the action of light is joined to a ſtronger reſiſting power in the ſmall parts of bodies, it may produce an acid, when to a weaker an alkali; theſe repreſenting the extremes whereof the middle are neutrals. The greateſt quantity of light with the ſtrongeſt reſiſtance, the ſtrongeſt acid, and the greateſt degree of light with little reſiſtance, alkali. Violent acids burn like metal, violent alkali like a flame.

12. As the rays of light of different refrangibility produce by our eyes the different ſenſe of colours, ſo the parts of bodies, having different reſiſting powers, may by our mouths and noſes produce in us the different ſen-

ſations of taſte and ſmell: for theſe two ſenſes are evidently nearly allied.

13. Since the motion of the planets, and of their ſatellites, is cauſed by light, emitted from the atmoſpheres of the ſun and planets, by a fermentation in their atmoſpheres, or it may be that the emiſſion of light cauſes this fermentation, then the planets and comets moſt diſtant from the ſun may have ſufficient heat in their atmoſpheres, for the production of animal life and vegetation.

14. It ſeems neceſſary that not only the body of the comets, but likewiſe their animals and vegetables, and their atmoſpheres, conſiſt of matter having a greater power of reſiſtance: for otherwiſe they could not reſiſt the ſtrong action of light, in their near approach to the ſun, without having their coheſion diſſolved, and their parts diſſipated; nor could they in their great removal from the ſun preſerve that heat and degree of motion, which may be neceſſary for the continuance of any mechanical ſyſtem in them. On conſidering the infinite degrees of reſiſtance that may be in matter, the exiſtence of animals and vegetables in the ſun is not impoſſible.

15. Fermentation ſeems to be the principal contrivance in nature, by which animal and vegetable life is begun and continued; and that all vegetation is both begun and continued by a proper fermentation in the earth, as well as in the body of the vegetable. In like manner animal life and health principally conſiſts in a proper and regular fermenta-

mentation in the ſeveral parts of the body, and ſickneſs in an irregular, exceſſive, defective, or vicious fermentation through the whole, or ſome parts of the body. Death neceſſarily follows a total ſuppreſſion of this fermentation.

16. I imagine, that fermentation cannot ariſe from any thing that is commonly called a mechanic cauſe; but that it gives the firſt principles of action to the mechaniſm of an animal or vegetable. Therefore, tho' the mechanic philoſophy be of great uſe in explaining the particular ſymptoms of a diſeaſe, and in the indications of the method of cure, it cannot reach to the firſt principles of life, or of health, or of diſeaſes. In my opinion the mechanic phyſicians give not the leaſt ſatisfactory account of the manner that medicines operate in an animal.

17. To inquire into the nature of fermentations, ſeems to be the moſt uſeful ſtudy one can apply himſelf to, as thereby the moſt uſeful arts to human ſociety, agriculture and medicine, are moſt likely to be improved to perfection. I would therefore earneſtly recommend it to thoſe who have leiſure and opportunity, to make experiments for that purpoſe; and in this the numerous electrical experiments lately made, may be of no ſmall uſe, for explaining the nature of fermentation. For as by the approach of an electric and non-electric, the firſt emits a great ſhock of light at once, ſo if there were many ſuch ſmall bodies, electrics and non-electrics, variouſly mixed, they might by

mutual impelling and repelling each other, produce a violent inteſtine motion, like that of fermentation, and all of them thereby emit a greater quantity of light: but I only propoſe this as a hint for further trial and experiments. I ſhall only add, that ſome electric experiments ſeem to prove, that electricity promotes animal and vegetable fermentation, and thereby its uſe in medicine may be diſcovered; eſpecially where the fermentation is only to be increaſed in one particular place, and which cannot be ſo certainly done by medicines taken inwardly, as by topical application. However this be, I am perſwaded, whoever can invent a proper ſeries of experiments, for diſcovering the nature of fermentation, and reaſon juſtly from them, he will diſcover many things of great uſe to mankind, of more general uſe than the diſcovery of a method to find longitude at ſea. Sir *Iſaac Newton*'s Optics is the beſt pattern for experimental philoſophy, and in my opinion diſcover more of his great genius than his *Principia* does, though both of them be moſt extraordinary performances.

CHAP. VII.

Of the intelligent Being, and of the Formation and Duration of the ſeveral Syſtems in the Univerſe.

1. THE elementary parts of matter, or the ſmalleſt parts of which matter is ſuppoſed or imagined to be compounded, muſt have one ſingle ſimple action, for it cannot be imagined that a complication of different actions can ariſe from one ſimple uncompounded thing. The alledging of it ſeems to involve a contradiction, as it ſuppoſes both a multiplicity and a unity at the ſame time. Thus we ſee, that the mixed rays of light, after they are ſeparated and become ſimple or ſingle, let them be reflected or refracted, or agitated in any manner, ſtill produce one ſimple colour, and always are in the ſame degree refrangible, as is demonſtrated in Sir *Iſaac Newton's* Optics.

These are not
even conceivable
without each othe

2. The elementary parts of matter act uniformly, neceſſarily and invariably, always in the ſame manner, and with the ſame degree of force. Whatever differences are in their actions, they ariſe from their different quantities, or from the oppoſite, or mixed, or complicated action of the ſame or different kinds of matter. For otherwiſe this action could not be the object of mathematical enquiry,

which with the greateſt certainty determines the actions of matter, from their ſeveral quantities, and from the ratios theſe quantities have to each other.

3. Nothing in the action of matter can induce one to think that its action proceeds from any ſenſe, perception, intelligence or will, or that ſenſe or will can be eſſential to matter, or that they are naturally involved or complicated with the actions of matter: for our ideas of the actions of matter are perfect and compleat, though it were ſuppoſed that ſenſe, perception, intelligence, or will, exiſted no where but in ourſelves.

4. Then ſince we cannot doubt of the exiſtence of ſenſe or perception, intelligence and will, they muſt be the actions, operations, or properties of ſome kind of being diſtinct from what is commonly called matter. There is nothing in the actions of motion, reſiſtance or elaſticity, that raiſes in our minds any idea of ſenſe, perception, intelligence or will; otherwiſe we could not conceive any machine, for example a mill, without at the ſame time conceiving that it may have ſome degree of ſenſe or perception, or intelligence or will.

5. There is nothing then in the idea of the actions of matter, by which the parts of it can form themſelves into any kind of regular ſyſtem, with any view to ſerve any purpoſe or end; neither is there any thing in any ſyſtem of matter that we know, which neceſſarily ſuppoſes an exiſtence of that ſyſtem, or without which we cannot

imagine matter to exiſt. We cannot conceive any thing eſſential to matter, whereby ſuch a quantity of matter (for example) muſt exiſt in that part of ſpace where the ſun now exiſts, that it ſhould contain ſuch a proportion of reſiſting matter, and ſuch another proportion of light: that the moſt reſiſting parts of matter ſhould be collected into the ſun; that it ſhould be of a globular figure, *&c.* Or why matter of a different power of reſiſting from what is in the planets, ſhould be collected into the comets; or why one part of matter ſhould be collected and placed in ſuch order, as to form an animal, another part in ſuch another order, as to form a vegetable or plant. If there be nothing in the actions of matter to do this, then it muſt be done by ſomething different from matter.

6. That the diameter of the equator of the ſun and planets is longer than their axes, cannot be cauſed by their rotation: for the rotation on their axes muſt preſuppoſe, that the diameters of their equators are longer than their axes, otherwiſe there can be nothing to determine the rotation to be on their axes, that is to ſay, on that particular axis. It cannot be done by chance, for chance is a non-entity, it is only our ignorance of the cauſe: again, if the greater length of the equatorial diameters were cauſed by the rotation, they muſt continually increaſe; for if they be ſuppoſed once to increaſe by the rotation, the velocity in the equator muſt likewiſe increaſe, and the gravitation, by which the parts are retain'd, decreaſe;

therefore the ſhape and figure of the planets is cauſed by ſome other agent, beſides the action of matter in the planets. If the equatorial diameter were not longer than the axis, the axis could not be inclined to the plane of the orbit, and thereby none of the planets could have the different ſeaſons of ſummer and winter; and thereby could not be repleniſhed with ſuch variety of animals and plants, whoſe conſtitutions require different climates and different ſeaſons; therefore the figure of the ſun and planets is ſhaped with a certain view to ſerve a certain purpoſe.

7. Suppoſe a planet were placed any where in the plane of the ſun's equator infinitely extended, there is nothing in the actions of the ſun or of the planet, in the action of light or reſiſtance, or in the reaction by the æther of theſe actions, to determine the firſt motion of the planets towards the eaſt rather than towards the weſt; this determination at firſt is abſolutely indifferent from thoſe cauſes, it muſt then proceed from ſome other cauſe, and which has a view or purpoſe in its actions: for if ſome of the planets moved to the eaſt and others to the weſt, their motions would be more frequently diſturbed, and become more irregular, by their coming more frequently into the neareſt diſtance one with another, than when all of them move from weſt to eaſt.

8. Again, the mean diſtances of the comets are ſo far from the ſun, that there can be nothing to determine their motion, to eaſt or weſt, to north or ſouth, or to

any intermediate point. Yet it is remarkable, that none of them move nearly in the plane of the ecliptic, if they did not paſs very near ſome of the planets, and by their great attraction, and by the heat of their atmoſpheres, occaſion great diſorder, if not deſtruction, among the planets.

9. All theſe diſcover foreſight, deſign, and purpoſe, of which innumerable other inſtances may be given, in every part of the univerſe that comes within our knowledge. The more knowledge we have of any thing, the more intelligence we diſcover in its formation; but I confine myſelf to my preſent ſubject. The more common inſtances have again and again been taken notice of by others.

10. It follows then, that the firſt formation of all kinds of material ſyſtems, the greateſt and the leaſt, was made by ſome intelligent being; that ſome being form'd, the grand ſolar ſyſtem, the more particular ſyſtem of this earth, and all the ſmall ſyſtems on it, whether animal, vegetable, or mineral: that the ſame intelligent being governs the great and the ſmall, each according to its nature, as is moſt conducive for the well-being of every individual, and of the univerſal ſyſtem of nature.

11. We have no idea of ſubſtances; we have as little knowledge of the ſubſtance of material beings as of intelligent ones; we have no idea of the thing in which the power of reſiſting, or of moving or of reacting, ſubſiſts, as little

as we have of the being in which intelligence ſubſiſts: but we may have ideas of the actions or operations of intelligence, as we have of the actions of matter, or as we have of motion or reſiſtance.

12. The eſſential or characteriſtic diſtinction between the material agent and the intelligent agent is this: the material agents act always uniformly, and in all directions, they have no power in themſelves to increaſe their force of action, or to determine it to one direction more than to another, all alteration in their action or in the direction of them is made by ſomething external, which for that reaſon is called an *efficient cauſe*, they have no will, purpoſe, view, or deſign in their action. But the intelligent being determines and directs its own actions, by the purpoſe, deſign, or view which it has, and therefore its actions are ſaid to be determin'd or directed by *final cauſes*, and this direction by final cauſes is called the *will*; therefore in all actions of intelligent beings, which are likewiſe called *moral actions*, the intention, purpoſe, or will, is principally to be conſidered. This is the guiding principle in morality, policy, and religion.

13. The actions of intelligent beings cannot be the object of mathematical inquiry. For quantity, and the ratios of quantities, is the ſole object of mathematics, but there can be nothing of quantity in deſign, intention, or will. Therefore any inquiry into the actions of an intel-

ligent agent muſt be on different principles, from what are uſed in an inquiry into the actions of matter. But frequently our ideas ariſe from the complicated actions of intelligent and material agents, in which caſe, a mixture of mathematical and metaphyſical principles become neceſſary in our inquiries.

14. The intelligent agent never acts in oppoſition or contradiction to the material agents; for if it did, nothing but confuſion, contradiction and abſurdity could enſue: and there could be no need or uſe of machinery, or of a certain order and diſpoſition of the parts of matter in the ſeveral ſyſtems, which compoſe the univerſe. But the intelligent either ſo diſpoſes of the parts of the ſyſtem, that their complicated actions ſhall ſerve the purpoſes, which the intelligent being has in forming of the ſyſtems: or where the action of the material agent is not determined, by any thing external to it, and its action is indifferent to any direction, in ſuch caſe the intelligent being gives the action ſuch direction, as beſt ſuits its own purpoſe; and this without any contradiction or oppoſition to the action of matter. This ſeems to be the caſe, in the voluntary actions of animals. It ſeems probable, that the æther contained in the nerves has its elaſticity equal through the whole length; and therefore, as thereby any action is eaſily and equally communicated from the internal to the internal extremity, or from the internal to the external [*],

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[*] Chap. II. par. 5.

the mind can direct or ſtop the reaction at pleaſure. But theſe things I ſee only with a very glimmering light. The manner of animal motion is among the dark things hereafter to be diſcovered. But this ſeems ſelf-evident, that perfect intelligence, or wiſdom, will not, and conſequently never can act in contradiction to the action of matter, otherwiſe any thing may be abſurd, and there would be no diſtinction between truth and falſhood.

15. The parts of every ſyſtem have ſome general reference or connection with one ſingle point, by which they become a kind of unity, or one ſyſtem. This reference is commonly, if not always, done by means of the æther. Thus all actions, by the increaſing elaſticity of the æther in its ſeveral diſtances from the ſun, are reacted to the ſun's center. In like manner all actions within the ſphere of the earth's ſyſtem, are all reacted to the earth's center. In the animal and vegetable ſyſtems, there is one part in the ſyſtem with which all the parts of the ſyſtem have ſome kind of communication. Whatever new part is added, ſo that it becomes united in the common reference, it becomes part of the ſame ſyſtem. When any part is ſo far removed, as to have no farther reference to or communication with the common or principal point, it is no longer a part of that ſyſtem. So that the identity of the ſyſtem does not conſiſt in the identity of the parts, nor in the identity of its place in ſpace, or with reſpect to other ſyſtems, but in the common reference of the parts

to

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to the ſame point. While the communication of action between the principal point and parts of the ſyſtem remain perfect, the ſyſtematical action, or health and life of the ſyſtem, remains perfect. When that communication is leſſened or diſordered, the ſyſtem becomes irregular or ſick. When this communication is cut off from any part, that part dies; when the communication in general ceaſes, the ſyſtem is diſſolved, and conſequently the union between the ſyſtem and the intelligent being ceaſes. For after this it is not properly a ſyſtem, when there can be no communication of action between the principal point and the whole ſyſtem, or between the ſyſtem and its parts, or between the ſyſtem and the intelligent being.

16. No time can be ſuppoſed, when no ſyſtem of matter did exiſt. For if it be ~~not~~ conſiſtent with perfect intelligence, or wiſdom, that material ſyſtems do exiſt, they cannot in any time be improper; or that if in wiſdom it be neceſſary that they do exiſt, it cannot be conſiſtent with wiſdom that they do not. This is to be underſtood as to ſyſtems in the univerſal; for as to this or the other individual ſyſtem, as any one or two bears no proportion to an infinite number, the exiſtence of any one individual cannot increaſe or diminiſh the fitneſs or unfitneſs of the whole. Therefore there may have been a time when the preſent ſolar ſyſtem did not exiſt, and the diſtance of time or duration from its firſt exiſtence to the preſent time may be

Excellent reasoning this! & to a fine purpose finite, and conſequently it may be of any quantity of finite duration to the preſent time.

17. That every particular ſyſtem has had a beginning, and will have an end, ſeems highly probable from this, that all the ſmaller ſyſtems, ſuch as thoſe of animals and vegetables, have a beginning and an end, and their durations are of various lengths: ſo likewiſe the great ſyſtems muſt alſo have their periods of exiſtence. For as the ſmall ſyſtems naturally ceaſe and are diſſolved, by the decay of that fermentation, which gives them action or life; ſo the great ſyſtems muſt neceſſarily ſuffer ſome ſuch decay. For example, the motion in this ſolar ſyſtem is continued and preſerved by the perpetual emiſſion of light; now this ſtore of light in the ſun muſt at laſt be ſpent, ſince there are no evident means continually to ſupply the perpetual waſte of light. When this defect happens, the conſequences are very evident, and that they do happen frequently in one or other ſolar ſyſtem, appears probable from the diſappearing of ſome of the fixed ſtars.

18. But it ſeems probable, that light or other waſte in the ſun may be ſupplied from the comets. For as the light of the ſun decays, ſome one or other of the comets, which approach nearer the ſun, may in their perihelion unite with the ſun, and thereby a new quantity of matter be added to it, and a new fermentation be raiſed. This may be the more probable, becauſe it is obſerved, nature has contrived means in all ſyſtems to recruit their waſte

for ſome time, till at laſt, the waſte being greater than the recruit, a total diſſolution enſues.

19. Nature, or more properly ſpeaking, the infinite intelligent *Archeus* has ordered ſo, that, ſince the ſeveral individual ſyſtems muſt in time fail, from their natural conſtitution, this defect is ſupplied by the generation of new and ſimilar ſyſtems, the conſtant method of doing which is by fermentation, under the direction of the intelligent agent. So, ſuppoſing that all the comets, planets, and other parts of the ſolar ſyſtem, by the failure of light in the ſun, ſhould at any time be united with the ſun; then a chaos or confuſed mixture of the heterogeneous parts of matter muſt enſue; and thereby an extraordinary fermentation, during which the *Archeus* forms a new ſolar ſyſtem, and a new heaven and a new earth may be produced. This conjecture ſeems to be confirmed by the appearing of new ſtars, and reappearing of ſome which before had diſappeared.

20. The duration of all the ſolar ſyſtems probably is infinite, in reſpect to the duration of any ſmall ſyſtem on this earth, whoſe period we know; and yet the duration of the ſolar ſyſtems may be infinitely ſmall, in reſpect to the duration of the univerſe.

The *Egyptian* prieſts, and *Pythagoras* from theſe, ſeem to have had ſpeculations of this ſort, which they delivered out in myſtic terms to the people, and explained clearly to the initiated only.

CHAP. VIII.

The Application of the preceding Theory of the Motion of the Planets, to the Motion of the Earth, or to the apparent Motion of the Sun, and compared with Mr. Flamſteed's *Obſervations.*

1. IT ſeldom happens that one man excels, both in theory and in practical obſervations: they ſeem to require a different genius, the one a lively imagination, the other accuracy and a conſtant application, from which a velocity of imagination is apt to draw one away, unleſs it be accompanied with great care and judgment. The imagination ſeldoms fails of ſtraying and being bewilder'd, in phyſical reſearches, if it be not kept within bounds by a ſtrict obſervance of the phænomena. The theorical writers in aſtronomy have generally given their rules ſo, that none but they, who are much converſant in the moſt abſtruſe parts of the mathematical ſciences, can underſtand them; therefore I think it may be of uſe, to explain the method which I have uſed in forming equations. It is eaſy and may be intelligible to thoſe who only know the principles of common arithmetic and ſpherical trigonometry; there-

by gentlemen of leiſure may have an opportunity, from the obſervations of others, to improve aſtronomy by way of amuſement. Some of the greateſt improvements in the ſciences have been made by them, who uſed them only as an amuſement; and this reaſon may be given for it, that a too cloſe application to one kind of ſtudy fetters and cramps the imagination.

2. I follow Sir *Iſaac Newton* in ſuppoſing the greateſt equation of the earth's orbit to be 1° 56′ 26″, though Mr. *Flamſteed* makes it 1° 56′ 0″. But I muſt obſerve, that this equation cannot be determin'd nearer than to five ſeconds, by the moſt accurate obſervers, with any inſtruments now in uſe; nor can the place of the equinoxes be determined with any certainty nearer than to one minute. After I ſhall have ſhewn the method which I uſe, of forming equations, with Mr. *Flamſteed's* obſervations, in ſuch a manner, as may be of uſe to thoſe, who are not much converſant in ſuch matters. There is another table of equations, formed to one degree for the greateſt equation, which I had compoſed for my private uſe, and which may be of uſe to others, where tables ready formed are wanting.

3. But before I proceed to ſhew the agreement between my calculations and Mr. *Flamſteed*'s obſervations, I ſhall ſhew the method of finding the latitude of any place, by an example of that of the obſervatory of *Greenwich*: and next inveſtigate the obliquity of the ecliptic, from

the ſun's greateſt and leaſt meridian altitudes. Nothing is wanting to make this method perfect, but a certain knowledge of the refractions, and of the ſun's parallax at all altitudes, ſo as to remove all doubts on theſe points, which it may be hoped Dr. *Bradley* will have the honour to perform.

4. I now proceed to the method I uſe in forming equations, viz. $1^\circ\ 56'\ 26'' = 419160'''$, $\frac{419160}{90} = 4657'''\cdot 333$, &c $\frac{4657'''}{90} = 51'''\cdot 748 =$ to the leaſt term of the arithmetical progreſſion $51\cdot 748 \times 2 = 1''\ 43'''\cdot 496 =$ to the difference of terms of the ar. progreſſion $\overline{4657\cdot 333 \times 2} - 51'''\cdot 748 = 2'\ 34''\ 22'''\cdot 918 =$ to the greateſt term of the arithmetical progreſſion.

	″	‴	°	′	″	‴
1	00	51 . 748	1	56	26	00
	1	43 . 496				51 . 748
2	02	35 . 244	1	56	25	08 . 252
	1	43 . 496			02	35 . 244
3	04	18 . 740	1	56	22	33 . 008
	1	43 . 496			04	18 . 740
4	06	02 . 236	1	56	18	14 . 268
	1	43 . 496			06	02 . 236
5	07	45 . 732	1	56	12	12 . 032
		&c.			07	45 . 732
			1	56	04 .	26 . 300
					&c.	

See Paragraph 19. of Chap. 2. and Paragraph 27. of Chap. 3.

5. To find the latitude of the obſervatory at *Greenwich*, from Mr. *Flamſteed*'s obſervations in the Prolegomena to the third volume of his Hiſtoria Cœleſtis, p. 114. *December* 1690.

	°	′	″
The leaſt diſtance of the pole ſtar from the zenith	36	09	25
Refraction add —			39
The true leaſt diſtance —	36	10	04
The greateſt diſtance of the ſame ſtar from the zenith	40	52	10
Refraction add —			47
The true greateſt diſtance	40	52	57
Leaſt diſtance —	36	10	04
Their ſum —	77	03	01
Diſtance of the pole from the zenith	38	31	30½
Altitude of the pole in 1690	51	28	29½
Nutation of the axis to be added, the ☽ aſcending node being near Aries —			9
The true latitude of the obſervatory —	51	28	38½

6. To find the obliquity of the ecliptic from Mr. *Flamſteed's* obſervations mentioned in the ſame place.

1690 *June* 10. Diſtance of the ſun's center from the zenith	27	59	10
Refraction add —			28
Parallax ſubſtract			05
	27	59	33
Altitude of the pole 1690	51	28	29½
	23	28	56½
June 10th. The diſtance of the ſun from tropic			4
Obliquity of the ecliptic	23	29	00½

The next is to diſcover the obliquity from the ſun's leaſt meridian altitude, for which purpoſe Mr. *Flamſteed* gives an obſervation of the 12th of *December* 1690, at which time the ſun had paſſed the ſolſtice; but, as my calculation of the ſun's place differs conſiderably from Mr. *Flamſteed*'s, I ſhall give mine at length that it may be the more eaſily examin'd.

	°	′	″	‴
The ſun's mean diſtance from the vernal equinoctial point in the beginning of 1690	9	20	23	55
His mean motion *December* 12th	11	11	02	02
The ſun's mean place *December* 12th	9	01	25	57
The diſtance of the ſun from the apogee beginning of 1690	3	07	33	00
The motion for *December* 12th to be added			1	00
The ſun's diſtance for *December* 12th from the apogee	3	07	34	00
The mean anomaly	5	23	51	57
Equation ſubſtract			15	17
True anomaly	5	23	36	40
The ſun's mean place	9	01	25	57
Firſt equation ſubſtract			15	56
Sun's place in his orbit	9	01	10	01
2d equation add			2	59
	9	01	13	00
Equation of time 26″ ſubſtract				01
Sun's true place in the ecliptic	9	01	12	59
		88	47	01

23	28	50	9.6003606
88	47	00	9.9999021
23	28	30	9.6002627

Then the ſun is only 20″ from the tropic, whereas Mr. *Flamſteed* computes his diſtance at 37′. Then

	°	′	″
Dec. 12th, 1690. Sun's meridian diſtance from the zenith	74	53	57
Refraction add		3	17
Parallax ſubſtract —			10
	74	57	04
Altitude of the pole 1690	51	28	29½
	23	28	34½
Sun's diſtance from the tropic *December* 12th			20
Obliquity of the ecliptic — —	23	28	54½

The errors in the diviſions of Mr. *Flamſteed*'s quadrant ſubject his obſervation to an uncertainty, ſometimes of 20 ſeconds as he himſelf allows, otherwiſe this might determine the difference between Mr. *Flamſteed*'s and Sir *Iſaac Newton*'s refractions at this altitude.

7. Mr. *Flamſteed* in his *Prolegomena*, p. 134, aſcertains the ſun's place in the ecliptic, by obſervations of the difference in time, wherein the ſun's center paſſes the meridian, and the ſame fixed ſtars paſs it at diſtant times of the year; which method has this advantage, that the previous knowledge of the latitude of the place, where the obſervations are made, or of the refractions, is not neceſſary. I ſhall ſhew how far my calculations and his obſervations agree in this method.

		h	′	″	R. aſcenſion. °	′	″
1690, *March* 7.	Calx Caſtoris paſs'd the meridian	6	12	48	93	28	00
	Procyon —	7	31	23	113	10	40
	Pollux —— —	7	34	37	113	59	00
Sept. 15.	Calx Caſtoris —	17	51	22	268	31	40
	Procyon — —	19	09	58	288	13	45
	Pollux ——	19	13	09	289	01	40

	Calx Caſtoris. °	′	″	Procyon. °	′	″	Pollux. °	′	″	
March 7.	93	28	00	113	10	40	113	59	00	
Sept. 15.	91	28	20	71	46	15	70	58	20	Compl. to 360
	184	56	20	184	56	55	184	57	20	Diſt. of ſun's R, A.

Then the mean difference of all theſe right aſcenſions is 184° 56′ 51″.

But in this method of determining the difference of the ſun's right aſcenſion, at diſtant times, by the difference in time, in which the fixed ſtars, and the ſun's center paſs the meridian, it is to obſerved, that an error of one ſecond in time, will produce an error of 15″ in the calculation; ſo that for example, if there were an error on the 7th of *March*, an error of one ſecond in the tranſit of the ſtar, and another of one ſecond the ſame day in the tranſit of the ſun's center, theſe two may produce an error of 30 ſeconds in the ſun's place in reſpect to the ſtars: and again, if there were the like errors on the 15th of *September*, they may produce an error of one minute, in the calculation of the difference of the ſun's right aſcenſion, as it is evident they have done in one from another of

Mr. *Flamſteed*'s obſervations. I ſhall try whether theſe errors can be reaſonably corrected; for which purpoſe it is to be obſerved, that if the obſervations were truly made, the differences between the right aſcenſions of the ſtars themſelves on both days muſt be nearly equal.

	h	′	″
On the 7th of *March* the difference in time between Calx Caſtoris and Procyon — —	1	18	35
But on the 15th of *September* 1″ too much	1	18	36
On the 7th of *March* between Calx Caſtoris and Pollux	1	29	49
But on the 15th of *September* 2″ too little	1	29	47

Then the mean difference will be one ſecond in time leſs than that before found, which will reduce it to 184° 56′ 36″.

By ſimilar obſervations of the ſame ſtars on the 14th of *March* he makes the difference 178° 36′ 00″ between the 14th of *March*, and 15th of *September*, but by making the corrections on the 14th of *March* as on the 15th of *September* it will only be 178° 35′ 40″.

It is farther to be obſerved, that an error may be produced by the plane of the quadrant's being inclined to the plane of the meridian, and differently inclined at theſe ſeveral days; which error is difficultly avoided: and laſtly, the equinoctial points cannot be determined from theſe obſervations alone, which however is neceſſary for determining the right aſcenſion with accuracy.

8. Theſe things being premiſed, let us ſee how far theſe obſervations agree with the calculations of the ſun's place, from the equation tables hereafter given.

The ſun's mean diſtance from the equinox beginning 1690	9	20	23	55
His mean motion *March* 7 —	2	05	03	10
The ſun's mean diſtance from the vernal equinox	11	25	27	05
The diſtance of the apogee from the vernal equinox in the beginning of 1690 — —	3	07	33	00
Its motion in two months — —				11
The diſtance of the apogee *March* 7 —	3	07	33	11
The mean anomaly — —	8	17	53	54
Equation add —— —		1	54	19
The true anomaly — —	8	19	48	13
The ſun's mean place ———	11	25	27	05
Equation add — —		1	54	56
Sun's place in his orbit —	11	27	22	01
Second equation ſubſtract —				20
Sun's place in the ecliptic ——	11	27	21	41
Equation of time 8′ 32″				21
Sun's true place in the ecliptic —	11	27	22	02
Diſtance from the equinox		2	37	58

	°	′	″	
C. S. of the obliquity of the ecliptic 1690	23	28	50	. 9 . 9624618
Tang. of ſun's diſtance from equinox	2	37	58	8 . 6625973
T. of ſun's right aſcenſion ———	2	24	54	8 . 6250591

Sun's

Sun's mean diſtance in the beginning of 1690	9	20	23	55
His mean motion *September* 15	8	14	17	49
Sun's mean place *September* 15	6	04	41	44
Place of the apogee beginning of 1690	3	07	33	00
Its motion in eight months				44
Place of the apogee *September* 15	3	07	33	44
Mean anomaly	2	27	08	00
Equation ſubſtr.		1	56	19
True anomaly	2	25	11	41
Sun's mean place	6	04	41	44
Equation ſubſtr.		1	56	06
Sun's place in its orbit	6	02	45	38
Second equation add				05
Sun's place in the ecliptic	6	02	45	43
Equation of time 8′ 07″ ſubſtract				20
Sun's true place in the ecliptic	6	02	45	23

	°	′	″	
C. S. of the obliquity of the ecliptic 1690	23	28	50	9 . 9624618
Tang. of ſun's diſtance from the equinox add	2	45	23	8 . 6825530
Tang. of ſun's right aſcenſion	2	31	42	8 . 6450148

	°	′	″
C. of ſun's right aſcenſion *March* 7	2	24	54
Right aſcenſion *September* 15	182	31	42
Difference of right aſcenſion	184	56	36

Which agrees with Mr. *Flamſteed's* obſervation when corrected as above; but without ſuch correction the greateſt equation ought to be made 8 ſeconds leſs.

The ſun's mean diſtance beginning of 1690	9	20	23	55
Mean motion to *March* 14	2	11	57	08
Sun's mean diſtance from the equinox	0	02	21	03
Place of the apogee	3	07	33	12
Mean anomaly	8	24	47	51
Equation add		1	55	58
True anomaly	8	26	43	49
Sun's mean place *March* 14	0	02	21	03
Equation add		1	56	16
Sun's place in his orbit	0	04	17	19
Second equation ſubſtr.				4
Sun's place in the ecliptic	0	04	17	15
Equation of time 5′ 48″ add				14
Sun's true place in the ecliptic	0	04	17	29

C. S. of the obliquity of the ecliptic 1690	23	28	50	9.9624618
Tang. of the ſun's diſtance from equinox	4	17	29	8.8752884
Tang. of the ſun's right aſcenſion	3	56	14	8.8377502

Sun's right aſcenſion *September* 15	182	31	42
March 14	3	56	14
Difference of ſun's right aſcenſion	178	35	28

By Mr. *Flamſteed*'s obſervations and calculations of the difference of the ſun's right aſcenſion from the fixed ſtars, the greateſt equation ought to be leſſened 16 ſeconds; but when corrected, as I think it ought to be, it ought to be leſſened only 6 ſeconds: but, as before obſerved, theſe obſervations are not of themſelves ſufficient to determine the greateſt equation.

9. Mr. *Flamſtead* makes the ſun's place from the tranſit of the ſtars on the 17th of *June* 1690, to be $03^s\,06^\circ\,02'\,24''$, by my calculations it is $03^s\,06^\circ\,02'\,20''$: ſo that by his obſervations in this method I have placed the apogee right, becauſe any error in placing the apogee is moſt ſenſible when the ſun is neareſt the apogee.

10. But, in my opinion, the ſun's place may be more certainly determined from its meridian altitudes, independently either of the latitude, or refractions, or obliquity of the ecliptic: for, if it be allowed, that the earth always moves in the plane of the ecliptic, as I think it generally is allowed, then whatever the latitude, or refractions, or obliquity be, the meridian altitudes between the vernal and autumnal equinoxes, at equal diſtances from the ſolſtices, or from the reſpective equinoctial points, muſt be equal: and as the refractions, any where above 60 degrees of altitude, do not differ above one ſecond in one degree, the refractions in two altitudes, which do not differ above one degree, cannot make a ſenſible difference. The juſtneſs then of any equations may with more certainty be examined by this method, when the meridian altitudes are taken accurately, than by the tranſits: for an error of five ſeconds in the altitudes, while the ſun is not above 30 degrees from the equinox, does not oc-

caſion a greater error in determining the ſun's place, than an error of one ſecond of time does in the tranſit either of the ſun or of the ſtar. I ſhall therefore next examine my calculations by Mr. *Flamſteed's* obſervations of the ſun's meridian altitudes in the year 1690, which I believe to be the moſt accurate of all his obſervations, as he choſe his obſervations of this year for fixing the longitude and latitude of the fixed ſtars.

From *Flamſteed's* Hiſtoria Cœleſtis, vol. 2d. 1690.

The ſun's diſtance from the equinox by calculation.

	°	′	″	
March 7	2	37	58	8 . 6621387
Obliquity in 1690	23	28	50	9 . 6003606
Declinat. *March.* 7	1	02	55	8 . 2624993

Sun's diſtance from equinox by calculation.

September 15	2	45	23 .	8 . 6820503
Obliquity 1690	23	28	50	9 . 6003606
Declination *September* 15	1	05	52	8 . 2824109

March 7, 1690.

Diſtance from zenith of ſun's upper limb	52	14	40
Under limb	52	46	50
	105	01	30
Diſtance of the ſun's center	52	30	45
Declination ſubſtr.	1	02	55
	51	27	50

September 15th.

	°	′	″
Diſtance from the zenith of the ſun's upper limb	52	17	35
Under limb	52	49	45
	165	07	20
Center	52	33	40
Declination	1	05	52
	51	27	48

Here they agree to 2 ſeconds.

March 14th.

	°	′	″	
Sun's diſtance from the equinoctial point	4	17	29	8 . 8740692
Obliquity	23	28	50	9 . 6003606
Declination	1	42	30	8 . 4744298

September 7th.

Sun's diſtance from the equinoctial point	5	05	22	8 . 9479765
Obliquity	23	28	50	9 . 6003606
Declination	2	01	32	8 . 5483371

March 14th.

	°	′	″
Diſtance of the ſun's upper limb	49	29	10
Under limb	50	01	20
	99	30	30
Center	49	45	15
Declination	1	42	30
	51	27	45

September 7th.

	°	′	″
Diſtance from the zenith of the ſun's upper limb	49	10	10
Under limb	49	42	05
	98	52	15
Center	49	26	7½
Declination	2	01	32
	51	27	39½

Agree to 5 ½ ſeconds.

April 14th.

Sun's Diſtance from the equinoctial point	34	37	52
Declination	13	05	13

Auguſt 7th.

Sun's diſtance from the equinoctial point	35	09	54
Declination	13	15	58

April 14th.

Diſtance of the zenith from the ſun's upper limb	38	06	35
Under limb	38	38	45
	76	45	20
Center	38	22	40
Declination	13	05	13
	51	27	53

Auguſt 7th.

Diſtance from the zenith of the ſun's upper limb	37	55	50
Under limb	38	27	55
	76	23	45
Center	38	11	52½
Declination	13	15	58
	51	27	50½

Here they agree to 2 ½ ſeconds.

May 7th.

	°	′	″
Sun's diſtance from the equinoctial point	56	50	09

July 15th,

Sun's diſtance from the equinoctial point	57	13	58

May 7th.

Diſtance from the zenith of the ſun's upper limb	31	43	00
Under limb	32	14	30
	63	57	30
Center	31	58	45
Declination	19	29	00
	51	27	45

July 15th.

Diſtance of ſun's upper limb from the zenith	32	09	00
Under limb	31	37	30
	63	46	30
Center	31	53	15
Declination	19	34	30
	51	27	45

No difference between theſe two,

June 2d.

Sun's diſtance from the equinoctial point	81	43	39

June 20th.

Sun's diſtance from the equinoctial point	81	06	03

June 2d.

	°	′	″
Diſtance from the zenith of the ſun's upper limb	27	58	35
Under limb	28	30	05
	56	28	40
Center	28	14	20
Declination	23	13	19
	51	27	39

June 20th.

	°	′	″
Diſtance from the zenith of the ſun's upper limb	28	01	05
Under limb	28	32	45
	56	33	50
Center	28	16	55
Declination	23	10	52
	51	27	47

Here the difference is 8 ſeconds. It is to be obſerved, that Mr. *Flamſteed*'s ſeveral obſervations of the meridian altitudes, on the 2d of *June*, all agree; but on the 20th one obſervation of the center makes it 15 ſeconds leſs than that which I have taken. I ſhall only add one pair of obſervations more, between the equinoxes and the winter ſolſtice.

February 14th.

	°	′	″
Sun's diſtance from the equinoctial point	23	34	37

October 6th.

	°	′	″
Sun's diſtance from the equinoctial point	23	31	16

February 14th.

Diſtance from the zenith of the ſun's upper limb	60	21	50
Under limb	60	53	45
	121	15	35
Center	60	37	47½
Declination	9	10	12
	51	27	35½

October 6th.

Diſtance from the zenith of the ſun's upper limb	60	20	25
Under limb	60	52	30
	121	12	55
Center	60	36	27½
Declination	9	08	58
	51	27	29½

Here the difference is 6 ſeconds. On the 6th of *October* one obſervation of the upper limb 60° 20′ 30″, and one of the center 60° 36′ 35″.

11. I have, in a conſiderable number of other places of the earth's orbit, compared, in this manner, the table of equations with Mr. *Flamſteed*'s obſervations, and I found them every where agree as much as in thoſe here given. This, in my opinion, fully proves the truth of this method of calculating, as much as can be done by his obſervations: ſince the calculations are every where within the extremes of his obſervations, where he has ſeveral obſervations of the meridian altitudes, while the body of the ſun is paſſing

the meridian; and ſuppoſing that the errors of his inſtrument are nearly the ſame, at altitudes nearly the ſame, as it is evident he himſelf thinks. If the ſun's mean diſtance from the vernal equinox in the beginning of 1690 were made five ſeconds leſs, *viz.* 9ˢ 20° 23′ 50″, the pairs of obſervations would in general agree more nearly, eſpecially near the equinoxes, at which diſtance any error in the mean diſtance is moſt ſenſible, and therefore it ought to have been taken ſo.

12. Sir *Iſaac Newton* appears in every thing to be ſo careful and accurate, that I have followed him in making the greateſt equation of the earth's orbit 1° 56′ 26″: but, if the errors of Mr. *Flamſteed*'s obſervations never exceed 20′, this equation ought to be made 26 ſeconds leſs, as Mr. *Flamſteed* makes it, that is, ſuppoſing either Mr. *Flamſteed*'s or Sir *Iſaac*'s tables of refractions to be near the truth. I deſigned to have added a general table, to facilitate the occaſional calculations, that any curious perſon may be deſirous to make, for determining the place of any of the planets at any time, any excentricity of their orbits being ſuppoſed; but ſome incidents have obliged me to break off all thoughts of this kind, without any inclination to renew them.

A Table of Equations of the Sun's apparent Motion.

Anom.		1ſt Equ.			Diff.		2d Eq.		Anom.	
Sign	Deg.	°	′	″	′	″	′	″	Deg.	Sign
3/9	00	1	56	26	0	01	0	00	0	3/9
	29	1	56	25	0	03	0	00	1	
	28	1	56	22	0	04	0	01	2	
	27	1	56	18	0	06	0	02	3	
	26	1	56	12	0	08	0	04	4	
	25	1	56	04	0	09	0	05	5	
	24	1	55	55	0	11	0	07	6	
	23	1	55	44	0	13	0	10	7	
	22	1	55	31	0	15	0	13	8	
	21	1	55	16	0	16	0	16	9	
	20	1	55	00	0	18	0	20	10	
	19	1	54	42	0	19	0	24	11	
	18	1	54	22	0	22	0	28	12	
	17	1	54	00	0	23	0	34	13	
	16	1	53	37	0	25	0	39	14	
	15	1	53	12	0	27	0	54	15	
	14	1	52	45	0	28	0	54	16	
	13	1	52	17	0	30	0	58	17	
	2	1	51	47	0	32	1	03	18	
	11	1	51	15	0	34	1	07	19	
	10	1	50	41	0	35	1	11	20	
	9	1	50	06	0	37	1	24	21	
	8	1	49	29	0	39	1	32	22	
	7	1	48	50	0	41	1	40	23	
	6	1	48	09	0	42	1	48	24	
	5	1	47	27	0	44	1	56	25	
	4	1	46	43	0	46	2	04	26	
	3	1	45	57	0	47	2	12	27	
	2	1	45	10	0	49	2	21	28	
	1	1	45	21	0	51	2	30	29	
2/8	00	1	43	30	0	53	2	40	0	4/10
	29	1	42	37	0	54	2	49	1	
	28	1	41	43	0	56	2	59	2	
	27	1	40	47	0	58	3	09	3	
	26	1	39	49	0	59½	3	18	4	
	25	1	38	49	1	01	3	27	5	
	24	1	37	48	1	03	3	37	6	
	23	1	36	45	1	05	3	47	7	
	22	1	35	40	1	06	3	57	8	
	21	1	34	34	1	08	4	06	9	
	20	1	33	26	1	10	4	15	10	
	19	1	32	16	1	12	4	24	11	
	18	1	31	05	1	13	4	33	12	
	17	1	29	51	1	15	4	42	13	
	16	1	28	36	1	17	4	51	14	
1/7	15	1	27	19½			5	01	1	4/10

Anom.		1ſt Equ.			Diff.		2d Eq.		Anom.	
Sign	Deg.	°	′	″	′	″	′	″	Deg.	Sign
1/7	15	1	27	19½	1	18	5	01	15	4/10
	14	1	26	01	1	20	5	09	16	
	13	1	24	41	1	22	5	17	17	
	12	1	23	19	1	24	5	25	18	
	11	1	21	55	1	25	5	33	19	
	10	1	20	30	1	27	5	40	20	
	9	1	19	03	1	29	5	47	21	
	8	1	17	34	1	31	5	53	22	
	7	1	16	03	1	32	5	59	23	
	6	1	14	31	1	34	6	05	24	
	5	1	12	57	1	36	6	10	25	
	4	1	11	21	1	37	6	14	26	
	3	1	09	44	1	39	6	18	27	
	2	1	08	05	1	41	6	22	28	
	1	1	06	24	1	43	6	26	29	
1/7	00	1	04	41	1	44	6	29	0	5/11
	29	1	02	57	1	46	6	31	1	
	28	1	01	11	1	48	6	32	2	
	27	0	59	23	1	50	6	32	3	
	26	0	57	33	1	51	6	31	4	
	25	0	55	42	1	53	6	30	5	
	24	0	53	49	1	55	6	28	6	
	23	0	51	54	1	56	6	25	7	
	22	0	49	58	1	58	6	21	8	
	21	0	48	00	2	00	6	16	9	
	20	0	46	00	2	02	6	11	10	
	19	0	43	58	2	03	6	04	11	
	18	0	41	55	2	05	5	56	12	
	17	0	39	50	2	07	5	48	13	
	16	0	37	43	2	08	5	37	14	
	15	0	35	35	2	10	5	27	15	
	14	0	33	24	2	12	5	15	16	
	13	0	31	12	2	14	5	01	17	
	12	0	28	59	2	15	4	46	18	
	11	0	26	43	2	17	4	30	19	
	10	0	24	26	2	19	4	13	20	
	9	0	22	07	2	20	3	53	21	
	8	0	19	47	2	22	3	35	22	
	7	0	17	24	2	24	3	13	23	
	6	0	15	00	2	26	2	50	24	
	5	0	12	35	2	28	2	26	25	
	4	0	10	07	2	29	2	00	26	
	3	0	07	38	2	31	1	31	27	
	2	0	05	07	2	33	1	02	28	
	1	0	02	34	2	34	0	32	29	
0/6	00	0	00	00			0	00	00	6/12

The uſe of the preceding table evidently appears, from the examples in the laſt chapter. It may be ſufficient to obſerve, that the firſt equation is to be added, in the earth's motion from the perihelion to the aphelion, to the mean anomaly to find the true anomaly; and to his mean motion to find his true place in his orbit: and the ſame equations to be ſubſtracted, in the earth's motion from the aphelion to the perihelion.

The ſecond equation for reducing the ſun's place in his orbit to the ecliptic, is to be ſubſtracted from the ſun's place in his orbit, while the earth moves from the perihelion to the aphelion: and to be added to the ſun's place in his orbit, while the earth moves from the aphelion to the perihelion.

The equations correſponding to minutes and ſeconds of the mean and true anomaly, are found by the common rule of three, from the differences between the equations for each degree, which differences are given in the table.

AN

INTRODUCTION

TO THE

Doctrine of FLUXIONS,

OR THE

Arithmetic of INFINITES:

IN ORDER

To aſſiſt the Imagination in forming Conceptions of the Principles on which that Doctrine is founded.

AN INTRODUCTION TO THE Doctrine of Fluxions, &c.

AS I found it difficult for me to form clear and diſtinct conceptions of the principles, on which the doctrine of *Fluxions*, or the *Arithmetic of Infinites* is founded, I reduced ſome years ſince my thoughts on that ſubject to writing; that I might thereby more attentively conſider them, and their congruity with what has been delivered by the author of that method.

Theſe difficulties I then attributed to the weakneſs of my own imagination and judgment; but having lately found that Biſhop *Berkley*, in his treatiſe called, *The Analiſt*, has declared to the world his opinion in the following words: "That Sir *Iſaac Newton*'s demonſtration of "the doctrine of fluxions is defective, and that he (Sir "*Iſaac*) was not quite pleaſed with it himſelf. —The

" various inconſiſtent accounts, which this great author " gives of his *momentums* and his *fluxions*, may convince " every intelligent reader, that he had no clear and ſteady " notions of them, without which there can be no de" monſtration." The biſhop declares " this to be his " opinion, after he had taken as much pains as (he ſin" cerely believes) any man living to underſtand that great " author, and to make ſenſe of his principles." The biſhop likewiſe adds what follows: " that having freely " converſed with mathematicians of all ranks, and ſome of " the ableſt profeſſors, as well as made it my buſineſs to " to be informed of the opinion of others, being very " deſirous to hear what could be ſaid, towards clearing my " difficulties, or anſwering my objections; I do ſolemnly " affirm (and ſeveral of themſelves know it to be true) " that I found no harmony or agreement among them, " but the reverſe thereof, the greateſt diſſonance, and " even contrariety of opinions employed to explain what " after all ſeemed inexplicable.

Theſe things, I ſay, make me think, that the difficulties I found, were really in the ſubject, or in the method the authors had taken to convey their conceptions: ſince ſo great and acute a philoſopher as the biſhop is allowed to be, after ſo many years ſtudy, ſtill allows that he can form no conception of the principles, on which the method of fluxions is founded. I reſolved therefore to re-examine my former thoughts, with my pen in my hand, to aſſure myſelf that I had not ſlightly, or with too fond an opinion of my own conceptions, reſted ſatisfied in what I formerly had done. And now I remain ſo far convinced that Sir *Iſaac Newton* had clear and diſtinct conceptions of his own principles, that I cannot forbear wiſhing to have an opportunity of ſubmitting to the biſhop the manner in which I have conſidered this ſubject; for often x one receives a conception by one method of conveyance, which ~~they~~ could not by another. The reſult of my work is as follows:

x Synonymous with We or they or any but inelegant.

They. A Vulgarism

1. The ideas or conceptions of *finite* and *infinite*, ariſes from the comparing of two or more quantities together; for if they cannot be compared, or have nothing in common of which it may be ſaid both contain equal quantities, or one contains more or the other leſs, they cannot be ſaid either to be finite or infinite with reſpect to each other. Thus *ſpace* and *time* are neither finite nor infinite, with reſpect to each other, becauſe they have nothing in common, of which it may be ſaid the one has more or leſs than the other; or no part of the one can meaſure any part of the other, by which the ratio of the one to the other can be determined to be either finite or infinite, determinable or indeterminable, equal, greater or leſs.

2. When, on comparing two quantities, we evidently perceive that the leaſt, by multiplication or addition, can be made equal to, or greater than, the greateſt, or that the greateſt, by diviſion or ſubſtraction, can be made equal

to, or leſs than the leaſt: we ſay theſe two quantities are finite, in the moſt common acceptation of the word *finite*, both or one of theſe quantities, or their common meaſure at leaſt, by which we compare them, comes within the compaſs of our ſenſes, that is, it is neither too little nor too great for our conception to form a diſtinct image of it.

3. But when, on comparing two quantities, we clearly perceive, that the one is ſo much leſs or greater than the other, that no multiplication of, or addition to the leaſt, can make it equal to, or greater than the other, or no diviſion of, or ſubſtraction from the greateſt, can make it ſo ſmall as the leaſt; we ſay theſe two quantities are infinite with reſpect to each other. In the common acceptation of the word, that one of the two which comes within the compaſs of our ſenſes, or which may be meaſured by ſome quantity which falls within the compaſs of our ſenſes, or which may be meaſured by ſome determinable part of ſuch quantity, is *finite*, and the other we call *infinite*, whether it be the greateſt or the ſmalleſt.

Thus, time and eternity have ſomething in common, by which they may be compared, viz. the quantity of duration in each; but, in that compariſon, we evidently [redacted] that time, however multiplied or increaſed, can be equal to eternity. No number of hours, no number of years, no number of ages or centuries can equal eternity. Again, any determined ſpace and univerſal ſpace agree in extenſion; but no quantity of any determined ſpace, however multiplied or increaſed can equal univerſal ſpace. The ſpace which the ſolar ſyſtem occupies, can as little, by any multiplication, become equal to univerſal ſpace, as a cubic inch of ſpace can become equal to it. Yet it is evident, that an hour, a year, and an age, are all component parts of eternity; ſo likewiſe every finite or determined ſpace is a component part of univerſal ſpace.

4. In conſidering, therefore, infinite quantities, the things compared have a relation one to the other, or have ſomething between them greater in the one than in the other; but that the quantity of this thing in common in one exceeds the other ſo far, that the ratio of this exceſs is greater than any that can be determin'd, or that they never can be in any proportion the one to the other.

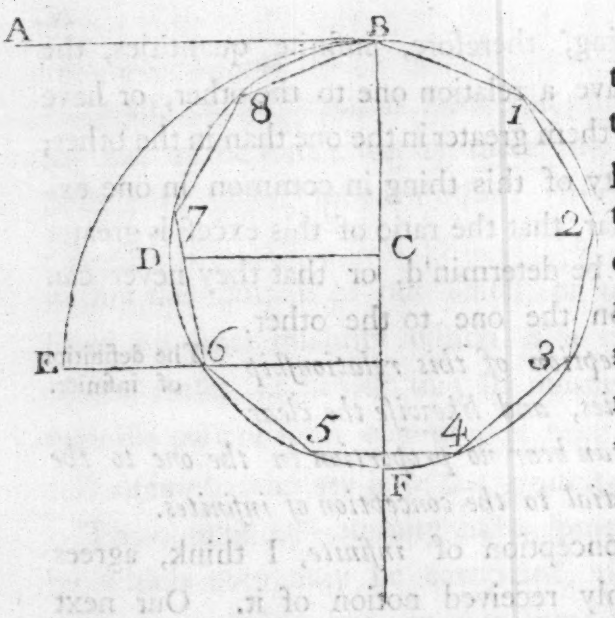
The definition of infinite.

So that, *The perception of this relationſhip between the quantities, and likewiſe the clear perception, that it can bear no proportion in the one to the other, are both eſſential to the conception of infinites.*

Thus far our conception of *infinite*, I think, agrees with the commonly received notion of it. Our next enquiry ſhall be, whether this conception agrees with that commonly received by mathematicians: or whether they have the ſame idea, when they ſay, one quantity is *infinite* in relation to another; or whether *infinite* in this ſenſe, can be a ſubject of mathematical ſcience or enquiry.

To ſhew that mathematicians have the ſame conception of infinites, I ſhall take the 16th prop. of the third book of *Euclid's* Elements, viz.

The angle comprehended between the tangent and the circumference is leſs than any acute angle; and the angle between the diameter and circumference is larger than any acute angle; and at the ſame time leſs than a right angle.



Therefore, I ſay, the angle of contact A B ~~D~~P, comprehended between the tangent and circumference, is infinitely ſmall, in reſpect to any (acute) right lined angle whatſoever. For if it be not, it may (by 2d above) by being multiplied by ſome finite number, become equal to a right lined angle; ſuppoſe equal to x; then divide the finite right lined angle x by that number, a right lined (acute) angle may by this diviſion be produced, equal to the angle of contact; which is abſurd, being contrary to what *Euclid* has demonſtrated. Since then no multiplication of the angle of contact can make it equal to any right lined angle whatſoever, it muſt (by the 3d) be infinitely ſmall.

But the ratio of the angle, at the point of contact made by the circumference, and any right line falling within it, as B C or 8 B to any other right lined angle *y*, will be the ſame with the ratio of the angle made by the tangent, and that ſtrait line falling within the circumference, to the angle *y*. If you ſay it is not, another right lined angle *z* may be made, which ſhall have the ſame ratio to the angle *y*, that the ſaid angle made by the circumference and a line falling within it has. But in this caſe, the difference between the angle made by the tangent and the line falling within the circumference of the angle *z*, will be equal to the angle of contact, and conſequently the angle of contact may be determined to be equal to a right lined (acute) angle, which is abſurd, being contrary to what *Euclid* has demonſtrated: and yet it is certain that the angle C B D is leſs than the angle A B C by the infinitely little difference A B D the angle of contact.——Therefore

5. The ſubſtraction or addition of an infinitely little or of any number of infinitely littles from or to a finite, does not alter the ratio of that finite to any other finite.

Now it is as evident a propoſition as any in geometry, that all the angles of any polygon, B, 1, 2, 3, 4, 5, 6, 7, 8, inſcribed in a circle, are equal to twice the number of right angles, which that polygon has ſides, after four right angles are deducted from the ſum or product: that is, calling the number of ſides *n*, and a right angle 90°, then $\overline{2n \times 90^\circ} - 360^\circ = S$, the ſum of the angles. Let then the ſides of the polygon be ever ſo many, while the number remains finite, the angle which any of theſe two ſides, as 8 B 1 makes, may be determined. It follows then, that the parts of a circumference, which compoſe the ſeveral bendings or angles thereof, muſt be infinitely ſmall, and the number of angles infinitely great, for till *n* become infinite, the ratio of any angle of the polygon to a right angle, or to any other angle, can always be determined; and if it can be determined, it muſt be either acute, right angled, or obſtuſe, and the angle of contact muſt conſequently be determin'd. But by conſidering, that the angles of a polygon grow more and more obtuſe as the number of ſides increaſe, when the number of ſides become infinite, the angle made by the two adjoining ſides, muſt be greater than any angle which bears any proportion to a finite angle; for the ſame reaſon, the length of the ſides muſt be leſs than that of any other determined or determinable line, or leſs than any line which bears any proportion to a determined or finite length. It is certain, at the ſame time, that the compounding parts or lines of every curve make an angle, or incline to each other, otherwiſe they muſt make a ſtrait line: ſo likewiſe the parts or lines which compoſe the curve, muſt be of ſome length, for what has no length can neither be finite nor infinite, nor even by an infinite addition or multiplication become of any length. Therefore

A circumference, or any other curve, is a polygon of an infinite number of angles, and of infinitely ſmall ſides: ſuppoſe then that from every angle of a circle, a line be drawn to the center of that circle, it will be divided into an infinite number of triangles: and as B *c* 1, 1 *c* 2, *&c.* the infinite number of angles meeting together at the center, taken all together, or their ſum, is equal to four right angles. Therefore

6. *An infinite number of infinites may be equal to a finite.*

Again, take any finite or determined part of this circle, ſuppoſe a ſemicircle, quadrant, ſextant, or octant, *&c.* I ſay, the number of theſe infinitely little triangles is infinite in every one of theſe, or in any other finite part of the circle, in the octant as well as in the whole circle. If you ſay they are not in the octant, for example, then the number muſt be finite: ſuppoſe 1000, then the number of the whole circle muſt be 8000, which is abſurd; for the number of the whole circle by ſuppoſition is infinite. Therefore

7. *No diviſion of an infinite by a finite, can reduce an infinite to a finite.*

8. *Finite and infinite terms cannot with propriety be join'd together in compoſing any ſum*; for they can have no ratio or proportion to each other, no number of additions

of the one can increaſe the other, nor no number of ſubſtractions leſſen it.

9. *One infinite may be double, treble, or quadruple, &c. of another infinite: or one infinite may be in any ratio to another infinite, which one finite can be in to another finite.*

10. *Finites may have an infinite variety of ratios to each other, before the one becomes infinite with reſpect to the other.*

In order to illuſtrate farther the doctrine of infinites, I ſhall return to the angle of contact. Another larger circle E B, or a ſmaller circle, may be made to touch the tangent in the ſame point; or an infinite number of circles, all of different diameters, may be made to touch a ſtrait line in the ſame point. Now it is evident that the angles of all theſe infinite polygons or circles, are all different, larger in the larger circles, and ſmaller in the ſmaller, and yet the ſmalleſt angle of the ſmalleſt circle is larger than any obtuſe right lined angle. Again, the angles of contact of all theſe ſeveral circles are all different, leſs at the larger circles, and greater at the ſmaller, and yet the greateſt is leſs than any acute right lined angle, and conſequently infinitely little. Thirdly, as theſe circles all bear the ſame proportion to each other as their reſpective diameters, and the infinitely little angles of contact will be reciprocally as their diameters, and laſtly, if the diameter of any circle B F be lengthened out beyond the circle at the end oppoſite to the tangent; this lengthened part of the diameter may be ſuppoſed divided into an infinite number of parts, ſince every line is infinitely diviſible; and a circle may be ſuppoſed drawn through each of theſe diviſions touching the tangent of the firſt circle in the ſame point, but the angle of contact with the firſt circle is infinitely ſmall. Therefore

11. *One infinitely ſmall may be divided into parts infinitely ſmaller; and theſe parts when compared with the finite, are ſaid to be infinitely infinite.*

By conſidering what has been ſaid of the angle of contact, I preſume the mind may extricate itſelf from many difficulties, which have puzzled ſome of the ableſt mathematicians, while they had not formed to themſelves juſt conceptions of the doctrine of infinites, in conſidering this and ſuch like theorems.

Geometry and arithmetic conſiſts in comparing quantities together, and in determining the ratio which we perceive in our mind one quantity has to another; in increaſing or leſſening, in multiplying or dividing theſe ratios at pleaſure, ſo as may beſt diſcover the ratio of ſome other quantity unknown, which we deſire to know; ſo that, *the quantities themſelves are not properly the objects of geometry or arithmetic, but the ratios of quantities.*

12. Since then (by the 8th) the ratios of infinites to each other, may be as thoſe of finites to each other, *infinites*

infinites are as properly the objects of geometry or arithmetic as finites are.

13. That part of arithmetick which ſhews the method of diſcovering the ratios of infinites, is called *the arithmetic of infinites,* or *the doctrine of fluxions.* I ſhall next conſider the principal rules of this method, and which chiefly diſtinguiſh it from the vulgar arithmetic.

14. *A finite, multiplied by an infinite, becomes infinite*; of conſequence a finite multiplied by an infinitely little, becomes infinitely little: as $4 \times \frac{1}{2}$, or by $\frac{1}{3}$, or by $\frac{1}{4}$, becomes 2, or $1\frac{1}{3}$, or 1; and if the fraction be infinitely little, the product muſt be ſo too; for the rule of multiplication is as unit to the multiplier, ſo the multiplicand to the product; therefore if the multiplier be infinitely leſs than unit, or any finite part of the mutiplicand, the product muſt be infinitely ſmall. For the ſame reaſon,

15. *An infinite multiplied by an infinite, becomes infinitely infinite*; that is, it will (ſuppoſing it to be infinitely ſmall) become as much leſs than any infinite term, to which before it bore ſome proportion, as that infinite term was leſs than any finite term.

16. *When any term is divided by an infinite, the term becomes infinite.* For the rule of diviſion is, as the diviſor to unit, ſo the dividend to the quotient: therefore if the

diviſor be infinitely ſmall, the quotient muſt be infinitely great; but if the diviſor be infinitely great, the quotient muſt be infinitely ſmall: the very contrary of what happens in multiplication, as the nature of the operations require.

17. *When any term of an equation, or of any ſum, becomes infinitely ſmall in reſpect to the others, it is properly rejected by the 7th*; for, ſince it can neither increaſe by addition, nor leſſen the ſum by ſubſtraction, it becomes inſignificant.

I come next to conſider the method geometers take to apply the doctrine of infinites to lines and figures. They ſuppoſe that any two lines, for example A . B and C . D are either divided into infinitely ſmall parts A———B, or elſe are generated in the ſame time by the C———D moving or flowing of the points from the points A and B, from the extremes A or C to the extremes B or D. If they ſuppoſe both to be infinitely divided, then the ratio of every infinitely little part in one, muſt be to every little part in the other as the one line is to the other.

18. If the lines be ſuppoſed both generated in the ſame time by the flowing of a point in each, and the time, in which they are generated, to be divided into infinitely ſmall portions or moments, the part of each line generated in this infinitely ſmall portion of time, is called the *naſcent augment* of that line; and the celerity with which each naſcent augment is made in the ſame moment or infinitely ſmall portion of time, is called the *fluxion* of that point; which

this cannot
be supposed
or conc—

which celerity or fluxion muſt be always as the lines themſelves generated in the ſame time. This laſt method of conſidering the fluxions, rather than the infinite ſmall parts of a line, I ſuppoſe is taken to avoid the diſputes, which otherwiſe would ariſe from the perplexed notions men commonly have of finite and infinite. For on the ſuppoſition of fluxions the things themſelves are not ſuppoſed to be conſidered, but the ratio only of the one to the other.

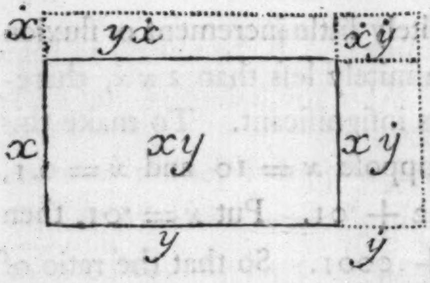
19. The mathematicians who ſuppoſe the infinitely ſmall parts of quantities, call them *differentials*; of the quantity x for example, by marking it $d\,x$, of the quantity y by $d\,y$, of z by $d\,z$. But the others, who explain their expreſſions by the ratio of the naſcent augment, expreſs the fluxion of the quantity x by $\dot{x}$, of y by $\dot{y}$, and of z by $\dot{z}$, &c.

They ſay naſcent augment, becauſe the line in an infinite ſmall part of time, let the velocity be what it will with which the point is ſuppoſed to flow, can acquire no determinable length, how ſmall ſoever it can be ſuppoſed, but has as it were only begun the ratio of celerity with which the line is ſuppoſed afterwards to be generated.

According to this laſt method of expreſſing, ſince $\dot{x}$ properly expreſſes the fluxion of x being always in the ſame ratio with the fluent or line generated, $2\,x\,\dot{x}$ will be the fluxion of $x\,x$: for $\overline{x+\dot{x}}|^2$ is $x\,x + 2\,x\,\dot{x} + \dot{x}\,\dot{x}$ ſubſtract the fluent $x x$, then the infinitely little increment or fluxion is $2 x \dot{x} + \dot{x} \dot{x}$; but $\dot{x} \dot{x}$ is infinitely leſs than $2 x \dot{x}$, therefore by 16th to be rejected as inſignificant. To make this the more eaſily conceived, ſuppoſe $x = 10$ and $\dot{x} = 0.1$, then $\overline{10 + 0.1}|^2 = 100 + .2 + {\cdot}01$. Put $\dot{x} = {\cdot}01$, then $\overline{10 + {\cdot}01}|^2 = 100 + {\cdot}02 + .0001$. So that the ratio of $\dot{x} \dot{x}$ to $\dot{x}$ continually decreaſes, as the ratio of $\dot{x}$ to x decreaſes, for as $1 : \dot{x} :: \dot{x} : \dot{x} \dot{x}$. If therefore the ratio of 1 to $\dot{x}$ be infinite, as upon the preſent ſuppoſition it is, the ratio of $\dot{x}$ to $\dot{x} \dot{x}$ muſt likewiſe be infinite: and in ſuch caſe the addition or ſubſtraction of $\dot{x} \dot{x}$ will not augment or diminiſh the value of $2 x \dot{x}$, ſo much as one drop of water will diminiſh or increaſe the quantity of the ocean.

For the ſame reaſon the fluxion of $x x x$ is $3 x x \dot{x}$, for $\overline{x + \dot{x}}|^3 = x^3 + 3 x x \dot{x} + 3 x \dot{x} \dot{x} + \dot{x} \dot{x} \dot{x}$. Now ſubſtracting the fluent $x x x$ the infinitely little increment, generated in the ratio of celerity with which the fluent moves, remains, viz. $3 x x \dot{x} + 3 x \dot{x} \dot{x} + \dot{x} \dot{x} \dot{x}$, but $3 x \dot{x} \dot{x}$ is infinitely leſs than $3 x x \dot{x}$, and $\dot{x} \dot{x} \dot{x}$ infinitely leſs than $3 x \dot{x} \dot{x}$; therefore both rejected as inſignificant in determining the ratio of its fluxion or increment. Thus in general the fluxion of x^n is $n x^{n-1} \dot{x}$.

If the fluxion of the parallelogram $x y$, whoſe ſides x and y are of different lengths, be deſired, then the fluxion



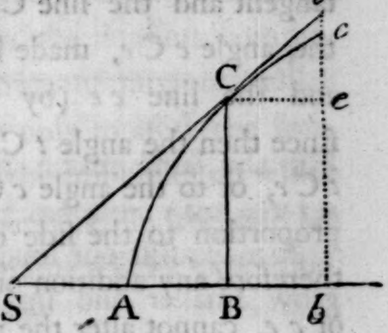
of xy is $x\dot{y} + y\dot{x}$, for $\overline{x+\dot{x}} \times \overline{y+\dot{y}} = xy + y\dot{x} + x\dot{y} + \dot{x}\dot{y}$, and ſubſtracting the fluent xy, and rejecting the infinitely infinite $\dot{x}\dot{y}$, the fluxion $y\dot{x} + x\dot{y}$ remains. For the ſame reaſon the fluxion of the paralelipepidon xyz is $xz\dot{y} + zy\dot{x} + xy\dot{z}$.

It may aſſiſt our imagination to obſerve, that as $\dot{x}$ or $\dot{y}$ grow ſmaller and ſmaller, the breadth of the parallelograms $y\dot{x}$ and $x\dot{y}$ decreaſes; and that when $\dot{x}$ and $\dot{y}$ become infinitely ſmall, $y\dot{x}$ and $x\dot{y}$ may be conſidered as lines or lengths only, or parallelograms of no breadth, and in ſuch caſe $\dot{x}\dot{y}$ muſt become a point of neither breadth nor length, that is, of no finite breadth or length, and therefore in calculation is taken as of no breadth or length, and can be of no uſe where the breadth and length or contents of a ſurface are inveſtigated.

In order to give a more compleat notion of fluxions, and to remove biſhop *Berkley*'s principal objection, I ſhall apply the general rules to one particular caſe.

A curve may be ſuppoſed to be generated in the following manner, *viz.* that the line B C move parallel to itſelf, along the line A B *b*, while a point C, in the line B C, paſſes from A, the beginning of the curve, along the line B C, towards C, in ſuch manner as to deſcribe the curve A C *c* by its motion. If this point flow in ſuch manner,

ner, that the intercepted diameter A B be always in ſome certain ratio to the ordinate B C, the nature of the curve may be determined. Thus putting A B $= x$ and B C $= y$, which two quantities are continually increaſing, as the point flows along the curve, and are therefore called fluents: and let d and p be put for any determined fixed quantities. Then if the point C flow in ſuch manner that in every part of the curve ☞ $px = yy$, the curve is a parabola, or if $dx - xx = yy$, then the curve is a circle, or if $\frac{d}{p}x - xx = yy$, the curve is an ellipſis, but if $dx + xx = \frac{d}{p}yy$, then the curve is an hyperbola.



Suppoſe that the finite part of the curve A C is already generated, and that the line B C in a moment, or infinitely ſmall portion of time, moves from B to *b*, and that, in the ſame time, the point C has flow'd from C to *c*, and generated the infinitely ſmall portion of the curve, or one ſide C *c* of the infinite polygon A C *c*. Draw the line C *e* parallel to A B *b*, and let S C *t* be a tangent to the curve at C meeting the produced diameter at S, and draw *b c* parallel to B C.

Now in this caſe the angle *t* C *c* repreſents the angle made by the tangent and the curve, or rather between the tangent and one ſide of the infinite polygon A C *c*, and is conſequently infinitely leſs than the angle made by the tangent and the line C *e* parallel to the diameter, or than the angle *c* C *e*, made by the ſide of the infinite polygon, and the line *c e* (by the 6th of 3d of *Euclid* as above) ſince then the angle *t* C *c* bears no proportion to the angle *t* C *e*, or to the angle *c* C *e*, the ſide or line *t c* can bear no proportion to the ſide or line *t e*, or to the line *c e*; and therefore any addition or ſubſtraction of *t c*, to or from *t e* or *c e*, cannot alter the ratio of *c e* or *t e* to C *e* or C *t*; not more (if ſo much) as the addition of an inch to the diameter of the earth's orbit can alter its ratio to the diameter of Mars, or Jupiter's orbit.

We have then the infinitely little triangle C *c e*, of which the ſide C *e* = B *b* equal to the fluxion or infinitely little increment of A B or of $x = \dot{x}$, and the ſide *e c* the fluxion of B C or of $y = \dot{y}$, I ſay, this infinitely little triangle is ſimilar to the triangle S C B; therefore *c e* : C *e* : : C B : B S; that is, $\dot{y} : \dot{x} :: y : \frac{y\dot{x}}{\dot{y}} =$ to the ſubtangent B S. And this formula will determine the ſubtangent of any curve whoſe interrupted diameter obſerves any certain ratio to its ordinate, and that ratio be given; but it can be of no uſe till a finite expreſſion be diſcovered, the terms of which are in the ſame ratio to each other with the terms of this infinite expreſſion.

The common method of finding theſe infinite numbers is to find the fluents of which theſe are fluxions; for the fluxions are always in the ratio of their fluents; but as the method of finding the fluents from the fluxions comprehends the greateſt part of the doctrine of fluxions, which is not my intention to deliver, but only to aſſiſt the imagination in forming conceptions of the principles on which that ſcience is founded. I ſhall therefore, in the caſe before us, ſhew another method of finding a finite expreſſion, in the ſame ratio with the infinite one.

For example, if from this infinite expreſſion $\frac{y\dot{x}}{\dot{y}}$ the ſubtangent of the parabola was deſired; take the equation $px = yy$, expreſſing the nature of the parabola, and find the fluxion of that equation, *viz.* $p\dot{x} = 2y\dot{y}$; then $p : 2y :: \dot{y} : \dot{x}$, and conſequently $\frac{2y}{p} = \frac{\dot{x}}{\dot{y}}$; then by equal multiplication $\frac{y\dot{x}}{\dot{y}} = \frac{2yy}{p}$, the ſubtangent of the parabola: that is, the ſubtangent of the parabola is always equal to twice the ſquare of the ordinate divided by the parameter. But if the ſubtangent of a circle be deſired, the fluxion of the equation for the circle is $d\dot{x} - 2x\dot{x} = 2y\dot{y}$, then $2y : d - 2x :: \dot{x} : \dot{y}$, and then the ſubtangent will be $\frac{2yy}{d - 2x}$, and by the like proceſs the ſubtangent (from this one formula) of any curve may be found, whoſe equation is given.

I ſhall now only add, that when we ſuppoſe two lines to be infinitely divided, and that the infinitely little parts are as the wholes, it is not thence neceſſary to ſuppoſe, that the number of infinitely little parts muſt be preciſely the ſame in both, in order to produce this equality of ratios of the parts and of the wholes. To think this neceſſary is an error one eaſily ſlips into, becauſe we are apt to imagine, that if the number of parts in one be greater than of thoſe in the other, the infinite little parts of the one will not have the ſame ratio to the infinite little parts of the other, which the wholes will have to each other, as is certainly true when the diviſion is in finite numbers. But if it be conſidered, that any finite number added to an infinite does not increaſe the ratio of that infinite; for example, if *a* and *b* are both ſuppoſed to be infinitely divided, but that *a* has 100 or 1000 of theſe infinitely little parts more than *b*, this will not (by the 5th) alter the ratio either of the wholes, or of any part of the infinitely little parts of the one to thoſe of the other. Therefore if we ſuppoſe 1000 or a million infinitely little parts, ſuch as the ſide C *c*, if the infinitely little triangle in the example above were added to the curve A C, this addition would not in any determinable rate augment the length of that curve.

From this I ſhall by way of concluſion obſerve, that the chief reaſon of the difficulty in forming proper conceptions of infinites is, that we endeavour to form diſtinct images of them in our minds, as we do of finites; whereas it is impoſſible to do this: for by their nature they bear no proportion to any finite image which we can form; they are either too big or too little for the compaſs of our imagination. We only can perceive the ratio which one infinite may have to another; and this we certainly may do, ſince it may be the ſame which one finite has to another; and of theſe ratios only the whole doctrine of infinites conſiſts. However difficult the forming the conception of infinites may be, they are however really ſuppoſed by all geometers, and the notion of them is coeval with geometry itſelf. The doctrine of fluxions is not abſolutely a new conception; the method of applying this conception is indeed a new invention, and a wonderful one it is, an eternal honour to the author of it. But, I ſay, the conception of infinites has always been ſuppoſed in geometry: for what is a point, a line, and a ſurface, as defined by geometers? A point, properly ſpeaking, is a quantity ſo ſmall, with reſpect to the other definite quantities with which it is compared, that it is ſuppoſed of no length, breadth, or thickneſs; that is to ſay, that it bears no proportion to the other determined or finite quantities: no addition or ſubſtraction of it can increaſe or diminiſh their ratio, either as to length, breadth, or thickneſs. So a line is where the breadth and thickneſs, and a ſurface where the thickneſs, are ſuppoſed ſo ſmall, that no addition of the breadth and thickneſs of the one, or of the thickneſs of the other, can increaſe or diminiſh the breadth or thickneſs of the quantity with which they are compared: for abſolutely and truly ſpeaking, a quantity of extenſion which has neither length, breadth or thickneſs (as a point is often defined) is a non-entity, and includes a contradiction in terms. So likewiſe length without breadth and thickneſs, or length and breadth without thickneſs, cannot be conceived. Therefore if according to the new method of fluxions x be length, z the breadth, and y the thickneſs of any finite or determined extended quantity, $x\,z\,y$ repreſents that quantity, and $x\,z\,\dot{y}$ is a ſurface of that quantity, $x\,\dot{z}\,\dot{y}$ a line of the ſame, and $\dot{x}\,\dot{z}\,\dot{y}$ a point. So that $x\,z\,\dot{y}$ a ſurface, is infinite with reſpect to the ſolid; $x\,\dot{z}\,\dot{y}$ a line, is infinitely infinite to the ſame; and $\dot{x}\,\dot{z}\,\dot{y}$ a point, is infinitely infinitely infinite. But if lines only be compared, and breadth and thickneſs be neglected, as not under the preſent enquiry, and $x\,z\,y$ repreſent the ſeveral lengths of thoſe lines, then $\dot{x}\,\dot{z}\,\dot{y}$ are points of thoſe lines. If ſurfaces be compared, both their length and breadth muſt be conſidered, but their thickneſs may be neglected, in which caſe they muſt be repreſented by two quantities, *viz.* x and z, and $x\,z$ repreſents a ſurface; in which caſe $x\,\dot{z}$ is a line of that ſurface, and $\dot{x}\,\dot{z}$ a point of the ſame.

Before I conclude, I muſt again obſerve, that though it be evidently demonſtrated, that ſeveral quantities may have an infinite ratio to each other, that is, that the one is infinitely greater than another, yet theſe infinite quantities, whether infinitely great or infinitely ſmall, are not the objects of our imagination; we can form no idea or picture of them in our mind, they are only the objects of our reaſon.

For though, from what has been already ſaid, we may be from our reaſon convinced, that a line may be ſtretched out to ſuch a length, that no finite line can bear any proportion to it, yet we can form no idea or picture of that infinite line.

Again, from reaſon we may perceive, that ſeveral lines may proceed from the ſame point, in ſeveral directions, and each of them be infinitely extended from that point; but then it does not follow, that theſe infinitely extended lines are all equal, or ought to be conſider'd as ſuch: but on the contrary, that they may have all the ſeveral proportions to each other, which any finite quantity has to another finite: that is, one of them may be twice as long as another, or thrice or four times as long, *&c.* The like is to be conceived of infinitely ſmall lines.

I cannot comprehend this.

Since then duration is conſidered as quantity, time as a finite or determinable quantity of duration, and eternity as an infinite quantity of duration, ſeveral things may have exiſted eternally, and yet the quantity of their ſeveral durations be very unequal, or in the ſame proportion that the exiſtence of things in time have to each other. For example, as we obſerve that ſome inſects exiſt but one day, others a week, others a month, or a year, *&c.* ſo ſome beings may exiſt eternally, and yet the quantity of their duration have the ſame proportion to each other, which the life of ſeveral animals whoſe exiſtence is in time have to each other; or it may not improperly be ſaid, that there are eternal days, eternal weeks, eternal months, or years, *&c.* and yet none of theſe eternal durations can have any proportion to the duration of God: for, as in conſidering the ratios of infinitely littles, $\dot{x}$ is ſuppoſed infinitely leſs than x, and x than $\dot{x}$, and ſo on, if $\dot{x}$ be ſuppoſed infinitely greater than x, $\ddot{x}$ muſt be infinitely greater than $\dot{x}$, and ſo on; conſequently the duration of God muſt be infinitely greater than any other infinite duration that can be ſuppoſed.

Non Intelligo

Non valet

It is then a moſt erroneous method of arguing, to ſay, that any thing or being does not exiſt, becauſe we can form no idea of that thing in our mind: for we may be moſt evidently convinced from reaſon, of the exiſtence of things, of which we can form no idea, as I think I have evidently ſhewn.

And now I have done all I can to prevent thoſe objections which occur from the prejudices we imbibe, by the faulty conceptions given us in our youth, while our preceptors attempted to teach what they themſelves did not underſtand.

FINIS.

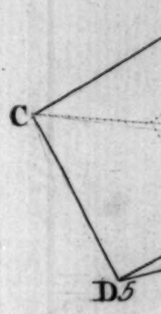
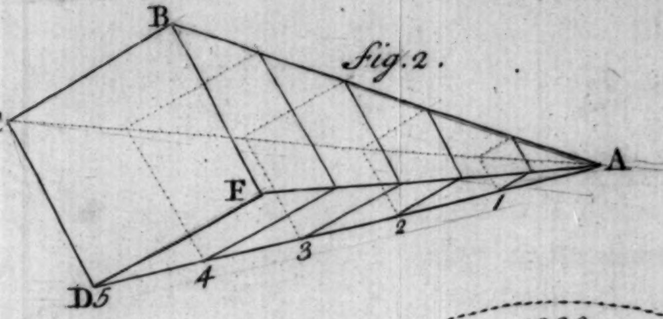


Fig.3



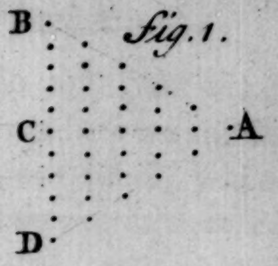
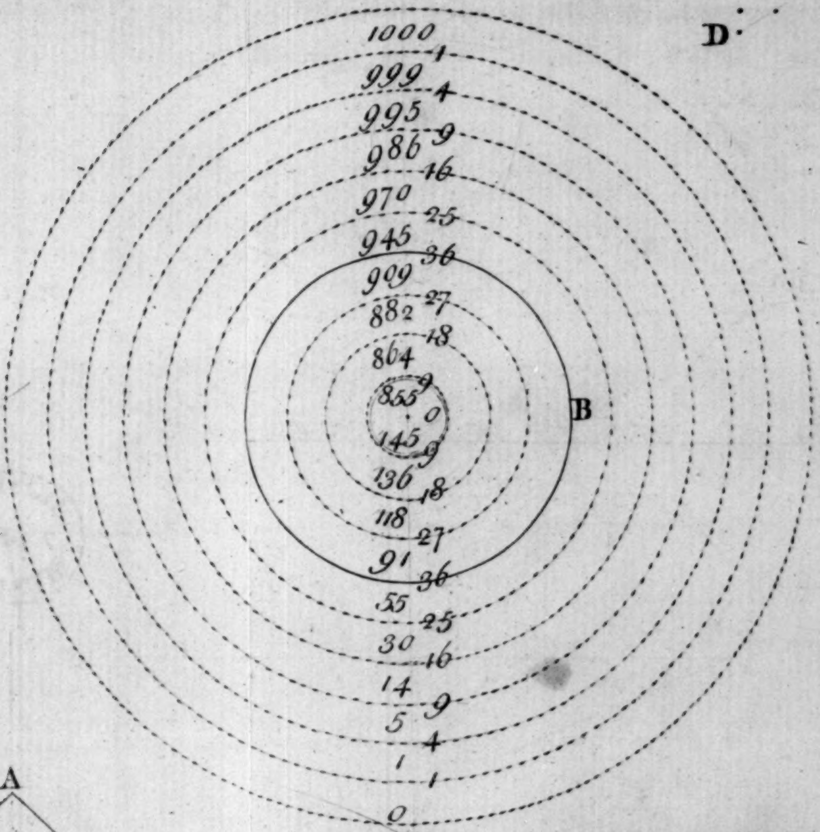
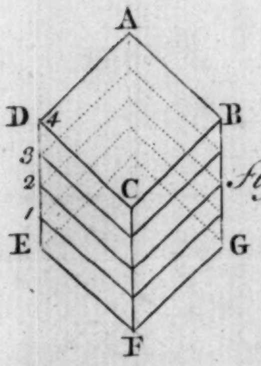


Fig. 3.





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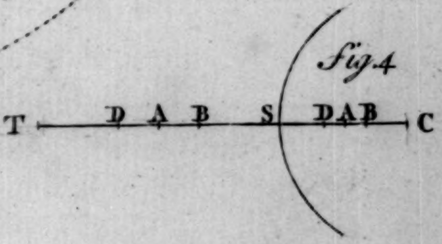


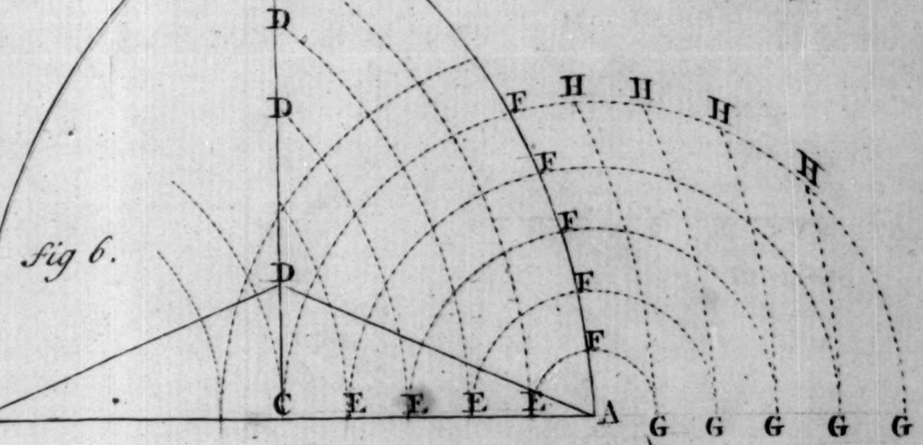


Fig 6.

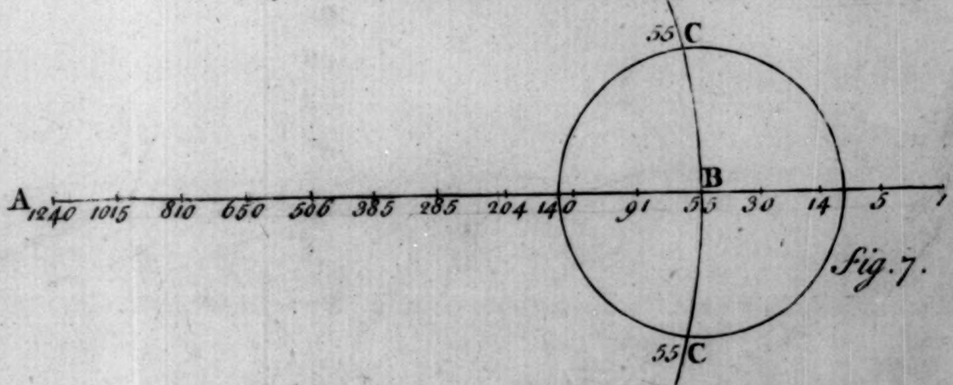
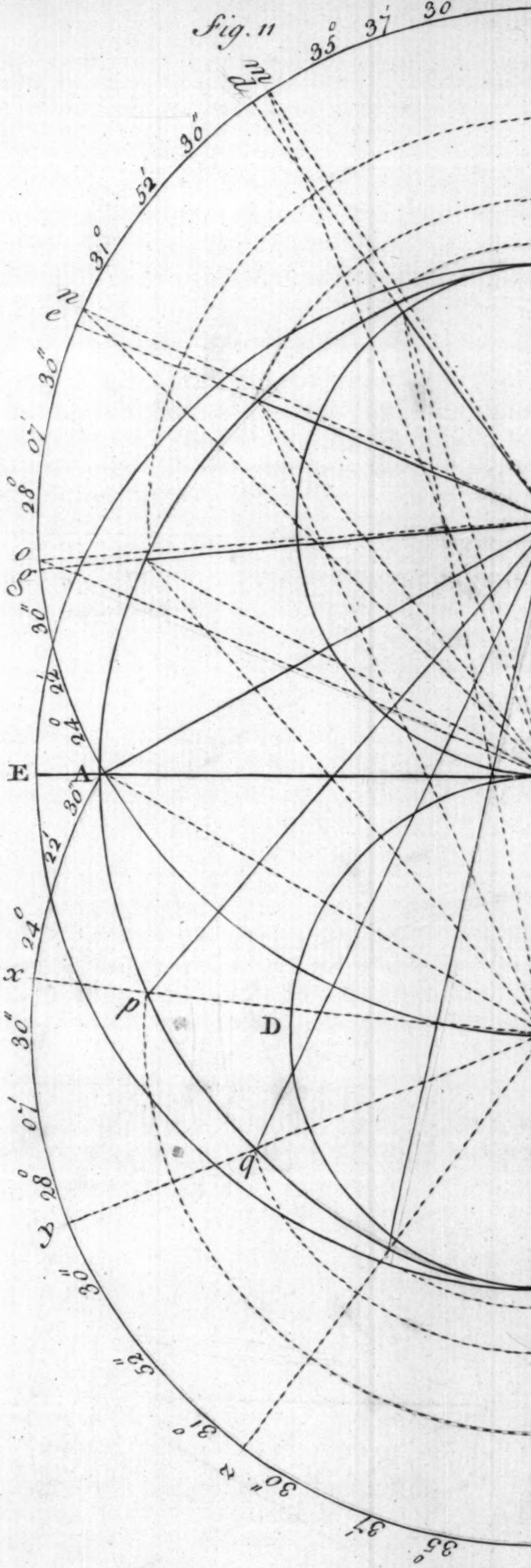


Fig. 7.

Fig. 11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z
30"
37'
35°

Fig. 12

f
7
5

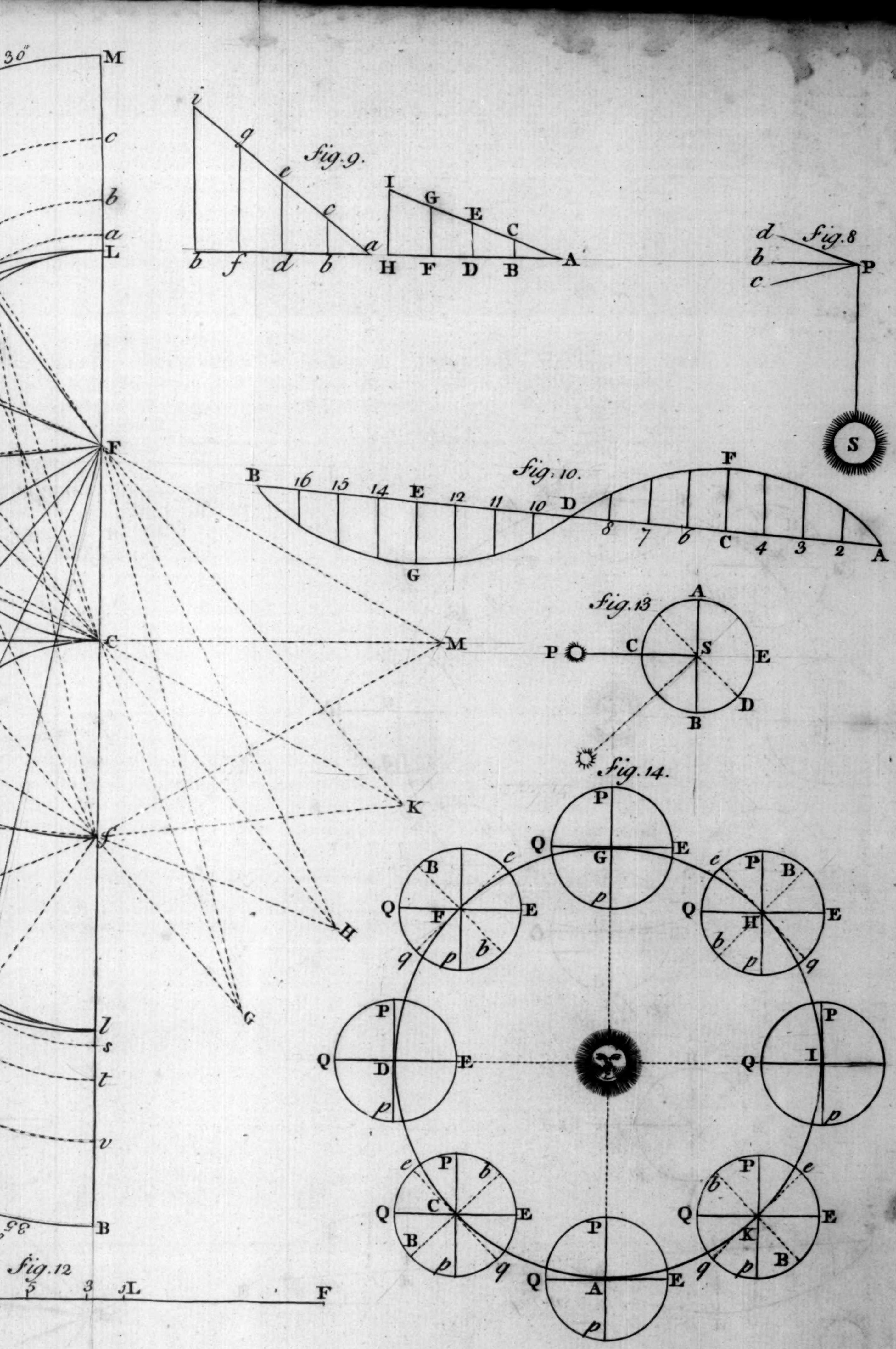

fig.9.

fig.8

fig. 10.

fig.13

fig. 14.

fig.12